8. FISSION AND FUSION

Induced fission – fissile materials

For $A \approx 240$, Coulomb barrier ≈ 5–6 MeV. If a neutron with zero kinetic energy enters nucleus to form compound nucleus, latter will have an excitation energy above its ground state = neutron's binding energy in that ground state. Example: zero-energy neutron entering ^{235}U forms ^{236}U with excitation energy 6.46 MeV >> fission barrier $\Rightarrow$ compound nucleus undergoes fission. To induce fission in ^{238}U requires neutron with kinetic energy > 1.4 MeV. Binding energy of last neutron in ^{239}U is 4.78 MeV < fission threshold of ^{239}U. Differences in binding energy of the last neutron in even-*A* (even-N/even-Z) and odd-*A* nuclei given by pairing term in SEMF. Explains why for odd-*A* nuclei $^{233}_{92}U$, $^{235}_{92}U$, $^{239}_{94}Pu$, $^{241}_{94}Pu$ fission may be induced by zero-energy neutrons, whereas $^{232}_{90}Th$, $^{238}_{92}U$, $^{240}_{94}Pu$, $^{242}_{94}Pu$ require an energetic neutron.

Commonly used fuel in reactors is uranium. (Natural uranium = 99.3% ^{238}U + 0.7% ^{235}U) σ_{tot} and σ_f, for neutrons incident on ^{235}U shown in Fig.8.1. Most important features are:

1. E < 0.1 eV, σ_{tot} for ^{235}U >> σ_{tot} for ^{238}U; fission fraction large (~ 84%) (rest mainly radiative capture with formation of excited state of ^{236}U)
2. 0.1 eV < E < 1 keV, cross-sections for both isotopes show prominent peaks due to resonant capture of neutron.
3. E > 1 keV, σ_f / σ_{tot} for ^{235}U still significant. In both isotopes σ_{tot} mainly due to contributions from elastic scattering and inelastic excitation of nucleus.

Widths of the low-energy resonances in ^{235}U cross-section are $\sim 1\text{eV}$, and compound nucleus formed at these resonances decay predominantly by fission $\Rightarrow$ fission takes place in a time of order $\tau_f = \hbar/\Gamma_f \approx 10^{-14}\text{s}$ after neutron absorption. Fission fragments are usually highly excited and quickly boil off neutrons. The average number of these so-called *prompt* neutrons for ^{235}U is $n \approx 2.5$. Decay products will decay by chains of β-decays and some of resulting nuclei give off further neutrons – *delayed* component – subject to mean delay of about 13s and may occur many years later $\Rightarrow$ biological hazard of radioactive waste.

Fission chain reactions

Define

$$k \equiv \frac{\text{number of neutrons produced in the } (n+1)\text{th stage of fission}}{\text{number of neutrons produced in the } n\text{th stage of fission}}$$

$k=1 \Rightarrow$ process is *critical* (sustained reaction can occur – ideal for operation of power plant); $k<1 \Rightarrow$ process is *subcritical* (reaction will die out); $k>1 \Rightarrow$ process is *supercritical* (energy will grow very rapidly, leading to an uncontrollable explosion)

Assume uranium is mixture of the two isotopes ^{235}U and ^{238}U in the ratio c:$(1-c)$, then average neutron total cross-section is $\bar{\sigma}_{tot} = c\sigma^{235}_{tot} + (1-c)\sigma^{238}_{tot}$ and *mean free path*, i.e. (mean distance neutron travels between interactions) is

$$\ell = 1/(\rho_{nucl}\,\bar{\sigma}_{tot})$$

where $\rho_{nucl} = 4.8\text{x}10^{28}$ nuclei /m^3 = nuclei density of uranium metal. For example, the average energy of a prompt neutron from fission is 2 MeV and $\sigma_{tot} \approx 7$ barns (Figs.8.1 and 8.2) so that $\ell \approx 3\text{cm}$. A 2 MeV neutron will travel this distance in about $1.5\text{x}10^{-9}\text{s}$.

Consider an *explosive release* of energy in a nuclear bomb, using ^{235}U ($c=1$). From Fig.8.1, neutron with this energy has probability of about 18% to induce fission in interaction with ^{235}U. Otherwise it will scatter and lose energy, so that the probability for a further interaction will be somewhat increased (cross-section increases with decreasing energy). If neutron does not escape outside the target, the most probable number of collisions it will make before inducing fission is about 6-7 $\Rightarrow$ moving a distance of 7cm in a time $t_p \approx 10^{-8}\text{s}$ before inducing fission. Not all neutrons will induce fission. (Some will escape from the surface and some will under radiative capture.) If probability that newly created neutron induces fission is *q*, each neutron on average leads to creation of $(nq-1)$ additional neutrons in time t_p. If there are $N(t)$ neutrons present at time *t*, then there will be

$$N(t+\delta t) = N(t)\left[1 + (nq-1)(\delta t/t_p)\right]$$

at $t+\delta t$. In the limit as $\delta t \to 0$,

$$\frac{dN}{dt} = \frac{(nq-1)}{t_p} N(t)$$

with solution

$$N(t) = N(0)\exp\left[(nq-1)t/t_p\right]$$

Thus number increases or decreases exponentially. For ^{235}U, number increases exponentially if $q > 1/n \approx 0.4$. If dimensions of the metal are << 7 cm, *q* will be small and chain reaction will die out. However, a sufficiently large mass brought together at $t=0$ will have $q>0.4$ (there will be neutrons present at $t=0$ arising from spontaneous fission) and since $t_p \approx 10^{-8}\text{s}$, explosion will occur in microsecond. For simple sphere of ^{235}U, critical radius at which $nq=1$ is about 8.7 cm; critical mass = 52 kg. Problem: energy released as assembly becomes critical will blow apart fissile material unless special steps are made to prevent this. One solution: fissile material (subcritical sphere of ^{239}Pu) surrounded by chemical explosives shaped so that when they explode the resulting shock wave *implodes* the plutonium to become supercritical.

Power from nuclear fission: nuclear reactors

Several distinct types of reactor. Main elements of *thermal reactor* shown in Fig.8.3 with core in Fig.8.4. Latter consists of fissile material (*fuel elements*), *control rods* and *moderator*. Natural uranium often used as fuel. A 2 MeV neutron has very little chance of inducing fission in ^{238}U. Much more likely to scatter inelastically, leaving nucleus in excited state. After couple of such collisions energy of neutron will be below threshold of 1.4 MeV for inducing fission. Neutron with its energy < 1.4 MeV has to find a nucleus of ^{235}U. Chances very small unless its energy has been reduced to < 0.1 eV, where cross-section is large. Before that happens it is likely to have been captured into one of the ^{238}U resonances. Thus, either fuel must be enriched with a greater fraction of ^{235}U (2%–3% is common in commercial reactors), or some method must be devised to overcome this problem. Role of *moderator:* (surrounds fuel elements – volume >> fuel elements) main purpose to slow down fast neutrons produced by elastic collisions in moderator. Absorption into resonances of ^{238}U is thereby avoided. Must have negligible cross-section for absorption and be inexpensive. (D_2O, or graphite used.)

In nuclear weapons, delayed neutrons of no consequence because the explosion will have taken place long before they would have been emitted; in power reactor they are of crucial importance, because the fuel rods can remain in reactors for several years. With delayed neutrons, each fission leads to $[(n+\delta n)q-1]$ additional neutrons (δn = number of delayed neutrons per fission). In steady state, $(n+\delta n)q-1=0$. By retracting or inserting *control rods* (made of cadmium – high absorption cross-section for neutrons) number of neutrons available to induce fission can be regulated. Key to maintaining constant *k* value of unity and therefore constant power output. Presence of delayed neutrons vital, because although lifetime of a prompt neutron may be as much as 10^{-3} s in reactor, rather than 10^{-8} s for pure ^{235}U, this would still be a very short time to change *q* and hence avoid a nuclear catastrophe! Reactor design ensures that $nq-1<0$, so can only become critical in presence of delayed neutrons. Thus time scale to manipulate control rods becomes that of delayed neutrons, (adequate). Important for thermal equilibrium that $dq/dT<0 \Rightarrow$ increase in temperature leads to fall in reaction rate. Rest is conventional engineering. From SEMF, fission of single ^{235}U nucleus releases ~200 MeV ($3.2\text{x}10^{-11}$ joules). (180 MeV is ‘prompt’ energy.) One gram of ^{235}U has about $6\text{x}10^{23}/235 \approx 3\text{x}10^{21}$ atoms; if fission were complete would yield total energy of about 10^{11} joules, (1 MW-day). $3\text{x}10^{6}$ times > yield obtained by burning 1 gram of coal. In practice only about 1% of the energy content of natural uranium can be extracted.

Fast breeder reactor – no large volume of moderator; no large density of thermal neutrons established. Proportion of fissile material ~ 20%; fast neutrons used to induce fission. Fuel (^{239}Pu) is obtained by chemical separation from spent fuel rods of a thermal reactor:

$$^{239}_{92}U \to ^{239}_{93}Np + e^- + \bar{\nu}_e \to \text{(3.6 days)}\ ^{239}_{94}Pu + e^- + \bar{\nu}_e$$

n = 2.96 for ^{239}Pu. Core: 20% ^{239}Pu and 80% ^{238}U (*depleted* uranium – obtained from spent fuel rods) surrounded by blanket of ^{238}U where more plutonium is made. Can produce more fissile ^{239}Pu than consumed (‘breeder’).

Major problem: generation of radioactive waste – no totally satisfactory solution available. In principle could “neutralise” long-lived fission fragments by using resonance capture of neutrons to convert to short-lived, or stable, nuclei. Example: ^{99}Tc (Technetium) has large resonant cross-section for neutron capture to ^{100}Ru (stable isotope). However, amount of radioactive waste very large and neutron energy has to be matched to particular waste material, which has to be separated and prepared $\Rightarrow$ increase cost of energy production.

Nuclear fusion: Coulomb barrier

B/A maximum at $A \approx 60$; slowly decreases for heavier nuclei. For lighter nuclei, decrease is much quicker, so in general lighter nuclei less tightly bound than medium size nuclei $\Rightarrow$ energy produced if two light nuclei fuse to produce heavier nucleus. Energy released comes from difference in binding energies of initial and final states (*nuclear fusion*). Energy released per fusion smaller than in fission. More than balanced by far greater abundance of stable light nuclei. Problem: Coulomb repulsion inhibits two nuclei getting close enough together to fuse. This is

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{ZZ'e^2}{R+R'}$$

Z and Z' = atomic numbers of the two nuclei, R and R' = radii ($R = 1.2A^{1/3}$ fm):

$$V_C = \left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)\frac{\hbar c\, Z Z'}{1.2\left[A^{1/3}+(A')^{1/3}\right]\text{fm}} = \frac{1}{137}\frac{197\,\text{MeV fm}}{1.2\,\text{fm}}\frac{Z Z'}{A^{1/3}+(A')^{1/3}}$$

setting (e.g) $A \approx A' \approx 2Z \approx 2Z'$, gives $V_C \approx A^{5/3}/8\,\text{MeV}$. With $A \approx 8, V_C \approx 4\,\text{MeV}$ has to be supplied to overcome Coulomb barrier. Only practical way is to heat mixture of nuclei to supply enough thermal energy. Necessary temperature estimated from $E = k_BT$, (k_B = Boltzmann’s constant). For energy of 2 MeV, temperature is $\sim 3\text{x}10^{10}$ K >> 10^7 K found in stellar interiors. Fusion actually occurs at lower temperature by combination of fact that energy distribution is Maxwellian (with high-energy tail), and tunnelling. (See Fig.8.5)

Stellar fusion

Energy of Sun due to nuclear fusion reactions, foremost is PPI cycle:

$$^1H + ^1H \to ^2H + e^+ + \nu_e + 0.42\,\text{MeV}$$

$$^1H + ^2H \to ^3He + \gamma + 5.49\,\text{MeV}$$

$$^3He + ^3He \to ^4He + 2(^1H) + 12.86\,\text{MeV}$$

Large energy release in the last reaction is because ^4He is doubly magic nucleus. Overall,

$$4(^1H) \to ^4He + 2e^+ + 2\nu_e + 2\gamma + 24.68\,\text{MeV}$$

Because temperature of Sun is $\sim 10^7$ K, all material is fully ionised (*plasma*) $\Rightarrow$ positrons produced annihilate with electrons to release further 1.02 MeV of energy per positron. Hence total energy released is 26.72 MeV. Of this, each neutrino will carry off 0.26 MeV on average (lost into space). Thus on average, 6.55 MeV of electromagnetic energy is radiated from Sun for every proton consumed in PPI chain. Another cycle that plays an important role in the evolution of some stellar objects, is CNO chain (Contributes ~ 3% of energy output of Sun.) Needs presence of any of the nuclei $^{12}_{6}C$, $^{13}_{6}C$, $^{14}_{7}N$ or $^{15}_{7}N$:

$$^{12}C + ^1H \to ^{13}N + \gamma + 1.95\ \text{MeV}$$
$$^{13}N \to ^{13}C + e^+ + \nu_e + 1.20\ \text{MeV}$$

$$^{13}C + ^1H \to ^{14}N + \gamma + 7.55\ \text{MeV}$$

$$^{14}N + ^1H \to ^{15}O + \gamma + 7.34\ \text{MeV}$$
$$^{15}O \to ^{15}N + e^+ + \nu_e + 1.68\ \text{MeV}$$

and

$$^{15}N + ^1H \to ^{12}C + ^4He + 4.96\ \text{MeV}$$

(^{12}C, for example, could be present because the ^{4}He produced in PPI cycle could fuse via $3(^{4}He)\rightarrow{}^{12}C + 7.27$ MeV.) Thus, overall:

$$4(^{1}H)\rightarrow{}^{4}He + 2e^{+} + 2\nu_e + 3\gamma + 24.68 \text{ MeV}$$

These, and other, fusion chains all produce electron neutrinos and from detailed models of Sun the flux of such neutrinos at surface of Earth can be predicted. Actual count rate << theoretical expectation. (*solar neutrino problem*). Possible solution: neutrino oscillations (see Sec 6) – only possible if neutrinos have mass.

Fusion reactors

PP reactions far too slow. Coulomb barriers for deuteron $^{2}_{1}H$ are same as for proton and the exothermic reactions

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}H + n + 3.27 \text{ MeV}$$

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{1}H + p + 4.03 \text{ MeV}$$

suggest deuterium might be suitable fuel for fusion reactor. Deuterium also present in huge quantities in sea water and is easy to separate at low cost. Even better reaction in terms of energy output is:

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + n + 17.62 \text{ MeV}$$

Advantages: heat of reaction is greater; cross-section is much larger. Principal disadvantage: tritium does not occur naturally (it has a mean life of only 17.7 years) – increases cost. Working energy where deuterium-tritium reaction has reasonable cross-section is about 20 keV, i.e. $3\text{x}10^{8}$ K. At these temperatures any material container will vapourise and so central problem is how to contain plasma for sufficiently long times for reaction to take place. Two main methods: magnetic confinement (ingredients are confined by electromagnetic fields); inertial confinement (small pellets of ‘fuel’ are imploded by bursts of pulsed laser beams). Although reactions have been observed, there is a very long way to go before a power plant could be built.

To achieve temperature T in deuterium-tritium plasma, there has to be an input of energy $4\rho_d(3k_B T/2)$ per unit volume. Here ρ_d is the number density of deuterium ions and the factor of 4 comes about because ρ_d is equal to the number density of tritium ions and the electron density is twice this, giving $4\rho_d$ particles per unit volume. Reaction rate in plasma is $\rho_d{}^2\Omega$, where Ω (measured in m^3s^{-1}) is related to the average cross-section for fusion reaction. If plasma is confined for time t_c, then, per unit volume of plasma,

$$L \equiv \frac{\text{energy output}}{\text{energy input}} = \frac{\rho_d{}^2\,\Omega\, t_c (17.6\ MeV)}{6\rho_d\, k_B\, T} \approx (10^{-19}\text{m}^3\text{s}^{-1})\,\rho_d\, t_c$$

where experimental value $\Omega \approx 10^{-22}$ has been used. For a useful device, $L > 1$, which implies

$$\rho_d\, t_c > 10^{19}\text{m}^{-3}\text{s}$$

This is the Lawson criterion $\Rightarrow$ either very high particle density or long confinement time, or both, is required.