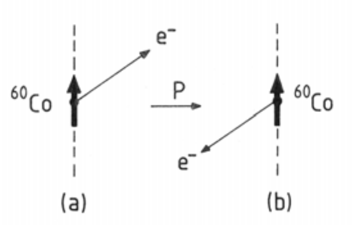
6. ELECTROWEAK INTERACTIONS

Charged and neutral currents

Neutral current reactions ⇒ mediated by Z^0 boson; *charged current* reactions ⇒ mediated by $W^\pm$ boson. *Electroweak interaction:* unifies weak and electromagnetic interactions, but unification only becomes manifest at very high energies; at low energies weak and electromagnetic interactions still clearly separated. Predicts existence of new spin-0 boson (*Higgs boson)* associated with origin of particle masses within the model.

Symmetries of the weak interaction

Will consider weak interactions alone and deduce general properties valid at all energies based on *parity* (*P*) and *charge conjugation* (C) operators. First direct demonstration of parity violation: study of ${}^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$ in magnetic field and cooled to 0.01K. At this temperature, interaction of magnetic moments of nuclei with magnetic field overcomes tendency to thermal disorder ⇒ nuclear spins align parallel to the field direction ⇒ cobalt-60 nuclei (partially) polarized. Parity violation established by observation of "forward-backward decay asymmetry", i.e. fewer electrons emitted in forward hemisphere than in backward hemisphere with respect to spins of the decaying nuclei. Follows because parity reverses particle momenta **p** while leaving orbital angular momenta $\mathbf{r} \wedge \mathbf{p}$ (and by analogy spin angular momenta), unchanged. In rest frame of decaying nuclei, effect is:



Parity invariance requires rates for both processes to be equal ⇒ equal numbers of electrons emitted in forward and backward hemispheres with respect to the nuclear spins – *contradicted by experiment*. Charge conjugation operator *C* changes all particles to antiparticles – also *not conserved in weak interactions.*

C-violation and *P*-violation in leptonic decays $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ and $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ of polarized muons. In rest frame of muon, angular distributions of final leptons are of the form

$$\Gamma_{\mu^\pm}(\cos\theta) = \frac{1}{2}\Gamma_\pm\left[1 - \frac{\xi_\pm}{3}\cos\theta\right]$$

θ = angle between muon spin direction and lepton momentum, $\xi_\pm$ = asymmetry parameters, $\Gamma_\pm$ = total decay rates (lifetimes $\tau_\pm^{-1} \equiv \Gamma_\pm$). *C* transforms μ^- decay to μ^+ decay ⇒ $\Gamma_+ = \Gamma_-$ and $\xi_+ = \xi_-$ (*C*-invariance). Parity preserves identity of particles, but reverses their momenta while leaving spins unchanged ⇒ $\theta \to \pi - \theta$, ⇒ $\Gamma_{\mu^\pm}(\cos\theta) = \Gamma_{\mu^\pm}(-\cos\theta)$ (*P*-invariance). Substituting for $\Gamma_{\mu^\pm}(\cos\theta)$ ⇒ $\xi_\pm = 0$ (*P*-invariance). Experimentally, $\mu^\pm$ lifetimes equal to very high precision ⇒ prediction for lifetimes satisfied; but $\xi_- = -\xi_+ = 1.00 \pm 0.04$ ⇒ both *C*-invariance and *P*-invariance violated.

Question: why do μ^+ and μ^- have same lifetime? Answer: *CP-conservation* (weak interaction is invariant under combined operation *CP*, even though both *C* and *P* are separately violated). *CP* operator transforms particles at rest to corresponding antiparticles at rest; *CP*-invariance ⇒ these states have identical properties. In muon decays, *P* changes θ to $\pi - \theta$, while *C* changes particles to antiparticles. Hence *CP*-invariance alone ⇒ $\Gamma_{\mu^+}(\cos\theta) = \Gamma_{\mu^-}(-\cos\theta)$. Substituting $\Gamma_{\mu^\pm}(\cos\theta)$, gives $\Gamma_+ = \Gamma_-$ (equal lifetimes) and $\xi_+ = -\xi_-$ (*CP*-invariance) in agreement experiment. (Only known violations of *CP* are very small effects that occur in the decays of some neutral mesons.)

Spin structure of weak interactions

(a) Neutrinos: Use *helicity states* (spin quantised along direction of motion of particle). Definitions for spin-1/2 particle: ν_L means left-handed neutrino (spin antiparallel to momentum), e_R right-handed electron etc. Fact: only left-handed neutrinos ν_L and right-handed antineutrinos $\bar{\nu}_R$ are observed in nature. Violates *C*-invariance (neutrinos and antineutrinos have identical weak interactions) and *P*-invariance (ν_L and ν_R have identical weak interactions), but compatible with *CP*-invariance:

$$\begin{tikzcd}
\boxed{\nu_L} \arrow[r, "P"] \arrow[d, "C"'] \arrow[dr, "CP"] & \nu_R \\
\bar{\nu}_L & \boxed{\bar{\nu}_R}
\end{tikzcd}$$

Helicity of neutrino first measured in reaction $e^- + {}^{152}Eu(J=0) \to {}^{152}Sm^*(J=1) + \nu_e$. Consistent with occurrence of left-handed neutrinos only. (Later experiments have shown that only right-handed antineutrinos take part in weak interactions.)

(b) Pions and muons: Spin dependence of V-A form. V denotes *proper vector*, (direction reversed by parity: example **p**) Because parity not conserved in weak interactions ⇒ corresponding weak charged current has another component whose direction is unchanged by a parity transformation – called an *axial-vector* (A) (example: $\mathbf{L} = \mathbf{r} \wedge \mathbf{p}$) Important result: in the limit that their velocities approach that of light, only left-handed fermions ν_L, e_L^- etc and right-handed antifermions $\bar{\nu}_R$, e_R^+ etc are emitted in charged current interactions ("forbidden" helicity states e_R^-, e_L^+ etc are suppressed by factors typically of order $m^2c^4/2E^2$, where *m* is the appropriate fermion mass.)

Illustrated by $\pi^+ \to \ell^+ + \nu_\ell$. In rest frame of pion, charged lepton and neutrino recoil in opposite directions ⇒ spins must be opposed (angular momentum conservation about decay axis). Since neutrino is left-handed ⇒ charged lepton must also be left-handed, in contradiction to the expectations for a relativistic antilepton. For positive muon this is unimportant, since it recoils non-relativistically so both helicity states are allowed. However, a positron *does* recoil relativistically ⇒ this mode is suppressed by $2(m_e/m_\pi)^2 = 2.6 \times 10^{-5}$. Thus positron decay mode is predicted to be much rarer than the muonic mode: measured ratio is $(1.218 \pm 0.014) \text{x} 10^{-4}$ (agrees with calculation). Helicity argument ⇒ muons emitted in pion decays are polarized (also agrees with experiment).

$W^{\pm}$ and Z^0 bosons

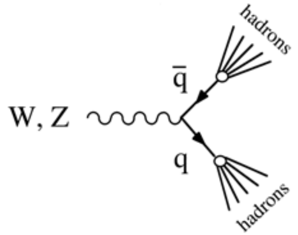
W^+ and W^- and Z^0 all discovered at CERN in 1983 in the reactions

$$\bar{p} + p \rightarrow W^+ + X^- \ , \quad \bar{p} + p \rightarrow W^- + X^+ \ , \ \text{and} \ \ \bar{p} + p \rightarrow Z^0 + X^0$$

$X^{\pm}$ and X^0 are arbitrary hadronic states allowed by conservation laws. Beams produced using a proton-antiproton collider (built specifically for this purpose). At the time it had proton and antiproton beams with maximum energies of 270 GeV each, (total centre-of-mass energy of 540 GeV). Two independent experiments were mounted, one (called UA1) shown schematically in Fig.6.11; example of the modern type of layered detector system. $W^{\pm}$ and Z^0 boson masses are:

$$M_W = 80.6 \text{ GeV / c}^2, \quad M_Z = 91.2 \text{ GeV / c}^2$$

lifetimes are about $3 \text{x} 10^{-25}\,\text{s}$. Dominant decays lead to jets of hadrons:

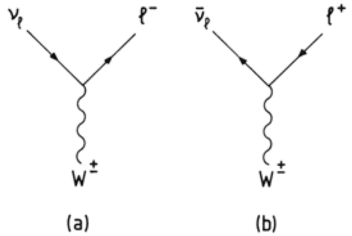


Also leptonic decays:

$$W^+ \rightarrow \ell^+ + \nu_\ell \ , \ W^- \rightarrow \ell^- + \bar{\nu}_\ell \quad \text{and} \quad Z^0 \rightarrow \ell^+ + \ell^- \ , \ Z^0 \rightarrow \nu_\ell + \bar{\nu}_l$$

In contrast to the zero mass photons and gluons, the $W^{\pm}$ and Z^0 are very massive particles ⇒ very short ranges, $R_W \approx R_Z \approx 2 \times 10^{-3}\,\text{fm}$.

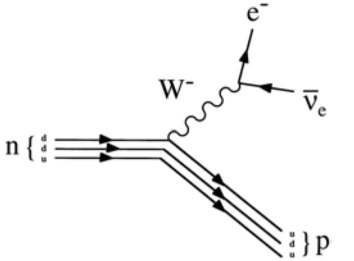
As in QED, Feynman diagrams for weak interactions are constructed from fundamental three line vertices. For lepton-quark (charged current) interactions:



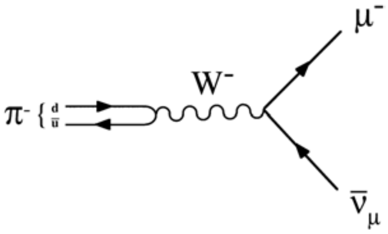
Strength of each vertex is given by (dimensionless parameter) $\alpha_w = g_w{}^2 / 4\pi\hbar c \approx 1/400$ for all three generations. Thus the weak interaction has a similar intrinsic strength to the electromagnetic interaction. Its apparent weakness in many low-energy reactions, is because the exchange bosons are heavy (effect of propagator).

Weak interactions of hadrons

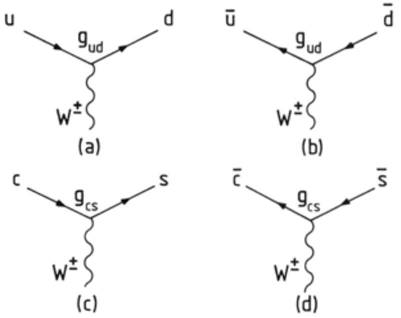
Understood in terms of basic processes in which $W^{\pm}$ bosons are emitted or absorbed by their constituent quarks, e.g. neutron decay: $d \rightarrow u + e^- + \bar{\nu}_e$:



and pion decay: $\pi^-(d\bar{u}) \rightarrow \mu^- + \bar{\nu}_\mu$:



Understood in terms of two ideas: *lepton-quark symmetry*, and *quark mixing*. (For simplicity, just consider two generations of quarks.) Lepton-quark symmetry: first two generations of quarks and first two generations of leptons have identical weak interactions ⇒ can obtain $W^{\pm}$-quark vertices from replacements $\nu_e \rightarrow u, \ e^- \rightarrow d, \ \nu_\mu \rightarrow c, \ \mu^- \rightarrow s$ in basic $W^{\pm}$-lepton vertices, leaving the coupling constant g_W unchanged. Resulting $W^{\pm}$–quark vertices:



$\Rightarrow$ fundamental processes $d + \bar{u} \rightarrow W^-$ and $s + \bar{c} \rightarrow W^-$ occur with *same* couplings g_W as corresponding leptonic processes, while processes $s + \bar{u} \rightarrow W^-$ and $d + \bar{c} \rightarrow W^-$ are forbidden, i.e. $g_{cs} = g_{ud} = g_W$. *But*: many decays forbidden in this simple scheme actually occur, albeit at a rate which is suppressed relative to the "allowed" decays. Example: $K^- \rightarrow \mu^- + \bar{\nu}_\mu$, which requires $s + \bar{u} \rightarrow W^-$ vertex that is not present in the above scheme.

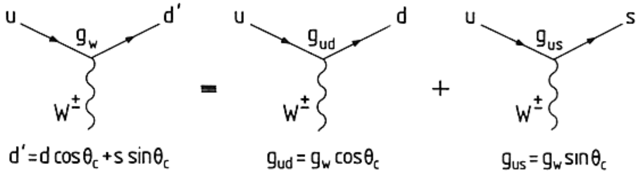
All suppressed decays can be successfully incorporated into theory by *quark mixing*: *d* and *s* quarks participate in the weak interactions via the linear combinations

$$d' = d\cos\theta_C + s\sin\theta_C \quad \text{and} \quad s' = -d\sin\theta_C + s\cos\theta_C$$

θ_C = *Cabibbo angle*. Lepton-quark symmetry now applies to the doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} c \\ s' \end{pmatrix}$$

Generates new vertices previously forbidden. Example: usW vertex required for $K^- \rightarrow \mu^- + \bar{\nu}_\mu$ arises from interpretation of $ud'W$ vertex shown:



Hypothesis enables theory and experiment to be brought into good agreement by choosing $\theta_C = 13^\circ$. Rates for previously "allowed" decays occur at rates which are suppressed by $\cos^2\theta_C \approx 0.95$, while previously "forbidden" decays are now allowed, but with rates suppressed by $\sin^2\theta_C \approx 0.05$. With third generation, mixing scheme becomes more complicated as we must allow for the possibility of mixing between all three "lower" quarks *d*, *s* and *b* instead of just the first two and more parameters are involved. For the first two generations, the changes introduced by this more complex mixing are very small.

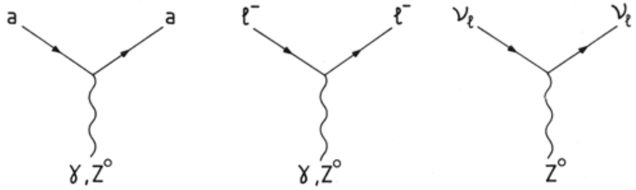
Neutral currents and the unified theory

Unified electroweak theory proposed mainly to solve problems associated with Feynman diagrams in which more than one *W* boson was exchanged. Such contributions expected to be small because higher order in weak interaction. Appears to be confirmed by experimental data. However, when these contributions are explicitly calculated, they are found to be infinite. In unified theory, problem solved when diagrams involving exchange of Z^0 bosons and photons are taken into account. These also give infinite contributions, but when all the diagrams of a given order are added together divergences cancel (!), giving well-defined and finite contribution overall. To ensure cancellation, theory requires relation between the weak and electromagnetic couplings (*unification condition*):

$$\frac{e}{2\sqrt{2}\varepsilon_0^{\ 1/2}} = g_w \sin\theta_W = g_z \cos\theta_W$$

weak mixing angle θ_W is given by $\cos\theta_W \equiv M_W / M_Z$; g_z = coupling constant which characterises strength of neutral current vertices.

Neutral current interactions accounted for in terms of basic Z^0-lepton vertices. Corresponding quark vertices can be obtained from the lepton vertices by using lepton-quark symmetry and quark mixing. Resulting set of vertices is:



$\Rightarrow$ neutral current interactions, like electromagnetic interactions, *conserve* quark numbers N_u, N_d, N_s ..., in contrast to the charged current interactions that do not conserve them.

Finally, in any process in which a photon is exchanged, a Z^0 boson can be exchanged as well. At energies that are small compared to the Z^0 mass, the Z^0-exchange contributions can be neglected compared to the corresponding photon exchange contributions, and these reactions can be regarded as purely electromagnetic to a high degree of accuracy. However, at very high energy and momentum transfers, Z^0-exchange contributions become comparable with photon exchange, and we are therefore dealing with genuinely electroweak processes which involve both weak and electromagnetic interactions to a comparable degree.

Illustrated by cross-section $e^+ + e^- \rightarrow \mu^+ + \mu^-$. Assume energy is large enough for the lepton masses to be neglected, then centre-of-mass energy *E* is the only quantity in the system that has dimensions. Because cross-sections have dimensions of an area, a simple dimensional argument gives the electromagnetic cross-section for one-photon exchange to be $\sigma_\gamma \approx \alpha^2 (\hbar c)^2 / E^2$. For Z^0–exchange with $E \ll M_Z c^2$, a similar argument gives the weak interaction cross-section $\sigma_Z \approx \alpha_Z^{\ 2} E^2 (\hbar c)^2 / (M_Z c^2)^4$ (neglecting factors of order unity). Thus one-photon exchange diagram dominates at low energies, and the cross-section falls as E^{-2}. Agrees with data (Fig.6.21). Relative importance of Z^0-exchange increases rapidly with energy and at about 50 GeV begins to make a significant contribution to total cross-section. At still higher energies, cross-section is dominated by a very large peak at an energy corresponding to the Z^0 mass (Fig.6.21). At this energy the low-energy approximation is irrelevant and real Z^0 bosons are formed followed by decay $Z^0 \rightarrow \mu^+ + \mu^-$. Finally, beyond the peak we once again regain the electroweak regime where both contributions are comparable and neither dominates.

Higgs boson

Neutral spin-0 boson whose existence is predicted by unified electroweak theory; not yet definitively observed. Required because of a fundamental symmetry called gauge invariance; requires that the spin-1 "gauge bosons" have zero masses if they are only bosons in the theory. OK for QED and QCD, but not for weak interactions. Gauge invariance also plays an important role in the unified electroweak theory, where it is needed to ensure the cancellation of the divergences that occur in individual Feynman diagrams. In this case result is stronger:

requires the fundamental particles – quarks, leptons and gauge bosons – *all* have zero masses if gauge bosons are the only bosons in the theory. This problem (*origin of mass*) overcome by assuming the various particles interact with new type of field (*Higgs field*). Two consequences: (1) gauge bosons can acquire masses without violating gauge invariance of the interaction; (2) there are electrically neutral quanta associated with Higgs (*Higgs bosons*) as there are quanta associated with the electromagnetic field.

The existence of the Higgs boson is the most important prediction of the standard model that has not been verified by experiment, and searches for it are a high priority. Problem: mass not predicted by the theory. But: couplings to other particles *are* predicted, and are essentially proportional to mass of the particle to which it couples. Higgs boson therefore couple very weakly to light particles (neutrinos, electrons, muons, u , d , s quarks) much more strongly to heavy particles ($W^{\pm}$ and Z^0 bosons, b and t quarks). Attempts to produce Higgs bosons thus made more difficult by the need to first produce very heavy particles to which they couple.

Failure to observe Higgs bosons in present experiments leads to limits on their mass. Best results come from $e^+ + e^- \to H^0 + Z^0$. Attempts were made to detect Higgs bosons by their decays to $b\bar{b}$ pairs where quarks would be observed as jets containing short-lived hadrons with non-zero beauty. The results were tantalizing. By the time LEP closed down in November 2000 to make way for another project, it had shown that no Higgs bosons existed with a mass less than $113.5\ \mathrm{GeV}/\mathrm{c}^2$; and had obtained some evidence for the existence of a Higgs boson with a mass of $115\ \mathrm{GeV}/\mathrm{c}^2$. Unfortunately, while signal is statistically likely to be a genuine result rather than a statistical fluctuation, the latter cannot be completely ruled out and it may well be several more years until new accelerators currently under construction are operational before we can be certain whether a Higgs boson exists.