5. QUARK INTERACTIONS: QCD AND COLOUR

(References to figures are those on the course website)

Colour

Quark model accounts successfully for hadron spectrum. (However: why only $3q$, $3\bar{q}$ and $q\bar{q}$ states observed?) But: contradiction with Pauli. Two quarks of same flavour in same spatial state must have same spin states (spins parallel). Example: $\Omega^- = sss$ baryon ($S=-3; J=3/2$) – all three quarks have spins parallel and no orbital angular momentum between them $\Rightarrow$ all three have same space and spin states $\Rightarrow$ overall wavefunction *symmetric* $\Rightarrow$ violation of Pauli principle (system of identical fermions must have a wavefunction overall *antisymmetric* under the interchange of any two particles) Q? how do the three *s*-quarks in the Ω^- differ? (Same problem for all hadron multiplets)

Solution: assume new degree of freedom called *colour*. Basic properties:

(a) Quarks exist in three different colour states (r, g, b for "red", "green" and "blue").

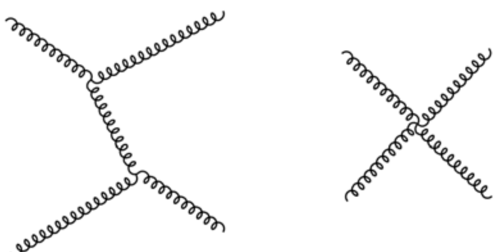
(b) Each state characterized by two conserved *colour charges* – I_3^C and Y^C – strong interaction analogues of the electric charge. These depend only on colour state, not on flavour. For multiparticle states, colour charges of individual states are added.

(c) Only states with zero values of colour charges are observable as free particles; (*colour singlets*) – called *colour confinement*.

Total wavefunction is product of spatial part $\psi(\mathbf{x})$, spin part χ, and colour wavefunction χ^C, i.e. $\Psi = \psi(x)\chi\chi^C$. Pauli principle interpreted as applying to *total* wavefunction including χ^C. Combined space and spin wavefunctions can then be symmetric under the interchange of quarks of the same flavour (to accord with experiment) provided the colour wavefunction is antisymmetric. Colour confinement is also 'solution' to why only qqq etc. (Unusual combinations like $qq\bar{q}\bar{q}$ and $qqqq\bar{q}$ could give rise to "exotic" hadrons but little really convincing experimental evidence for their existence.)

Quantum Chromodynamics (QCD)

Theory of strong interactions in standard model. Similar to QED, but important differences: gluons have zero electric charge, like photons, but unlike photons, which couple to electric charge, gluons couple to *colour* "charges" $\Rightarrow$ flavour independence of strong interactions ie all quark flavours have identical strong interactions (because they exist in the same three colour states r, g, b) $\Rightarrow$ equality of potentials in charmonium and bottomium systems; the near equality of the masses of charge states within multiplets etc. Non-zero colour charges for gluons leads to confinement of gluons and gluon-gluon couplings. Two lowest order contributions to gluon-gluon scattering.



(a) (b)

First is gluon exchange in analogy to gluon exchange in quark-quark scattering, second involves a "zero range" or "contact" interaction. These interactions could in principle lead to bound states of two or more gluon (*glueballs* – no very compelling evidence that they exist)

Gluon-gluon interactions have no analogue in QED, (photons couple to electrically charged particles and hence do not couple to other photons) $\Rightarrow$ properties of the strong interaction which differ markedly from those of the electromagnetic interaction – *colour confinement*, and *asymptotic freedom* = interaction gets weaker at short distances, (or as distance between the quarks increases, interaction gets stronger). Illustrated by static potential between a heavy quark and an antiquark. At short interquark distances $r \lesssim 0.1\,\text{fm}$, interaction is dominated by one-gluon exchange and we might expect Coulomb-like potential analogous to that arising from one-photon exchange in QED:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r}\hbar c \qquad (r \le 0.1\,\text{fm})$$

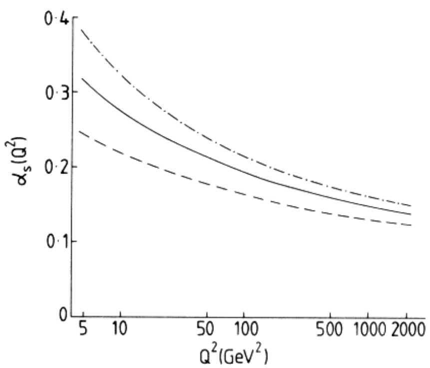
strong coupling constant α_s is analogous to the fine structure constant α in QED. Asymptotic freedom $\Rightarrow$ strength of interaction (and hence α_s), decreases with decreasing r, (for $r < 0.1\,\text{fm}$ variation is slight and can in many applications be neglected). For distances greater than 0.1fm, strength of interaction increases with increasing r, potential rises approximately linearly (*confining potential*) $V(r) \approx \lambda\, r$, (λ is of order $1\,\text{GeV fm}^{-1}$)

The strong coupling constant

Short-distance interactions associated with large momentum transfers $|\mathbf{q}|$ between the particles (cf de Broglie relation). Strength of interaction depends on the squared energy-momentum transfer $Q^2 \equiv \mathbf{q}^2 - E_q^2/c^2$; QCD coupling constant α_s is given by

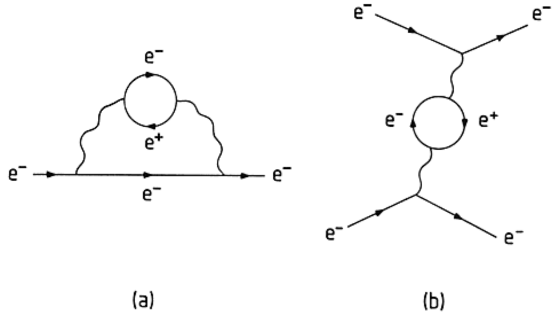
$$\alpha_s = \frac{12\pi}{(33 - 2N_f)\,\ell n\left(Q^2/\Lambda^2\right)}$$

for $Q^2 \gg \Lambda^2$; N_f = number of quark flavours, with $4m_q^2c^4 < Q^2$, Λ is a scale parameter (determined from experiment: $\Lambda = 0.2 \pm 0.1\,\text{GeV/c}$. Values of $\alpha_s(Q^2)$:



Asymptotic freedom

As quarks separate, energy stored in colour field increases until it becomes sufficiently large enough to create $q\bar{q}$ pairs. Eventually combinations of these will appear as physical hadrons – *fragmentation*. Emission and reabsorption of photons = *quantum fluctuation.* Second-order example with associated scattering:

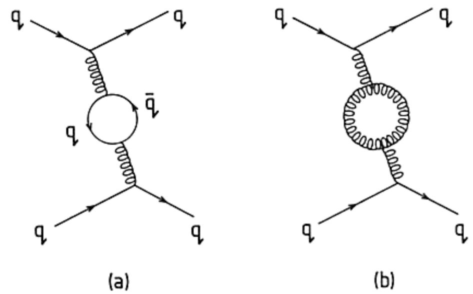


$\Rightarrow$ measurable consequences (*vacuum polarization effects*) $\Rightarrow$ QED coupling $\alpha_{em}(Q^2)$:

$$\alpha_{em}(Q^2) = \alpha(\mu^2)\left[1 - \frac{1}{3\pi}\alpha(\mu^2)\ell n\left(\frac{Q^2}{\mu^2}\right)\right]^{-1}$$

μ = low-energy value where we know value of α. If, for example, $\mu = 1\text{MeV/c}$ and $\alpha = 1/137$, i.e. the value of the fine structure constant as found from low energy interactions, then at the mass of the Z^0, $\alpha \approx 1/129$. Thus the electromagnetic coupling increases with energy-transfer, but only very slowly.

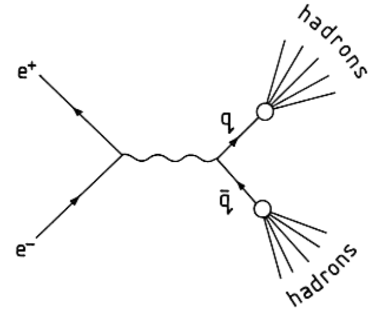
Examples in QCD:



First is analogous to virtual e^+e^- production in QED and also leads to a screening effect. Second diagram has no counterpart in QED and leads to a larger *antiscreening* effect $\Rightarrow$ interaction grows weaker at short distances, i.e. asymptotic freedom – α_s is given by a formula analogous to that for α_{em}, except the coefficient of the logarithmic term is different and, crucially, its sign is positive.

Jets and gluons

Studied in the reaction $e^+ + e^- \rightarrow$ hadrons at high energies using *colliding beam* experiments – ‘clean’ reaction, because the initial particles are elementary, without internal structure. For E_{CM} 15-40 GeV, dominated by the production of jets. Regarded as occurring in two stages: primary electromagnetic process $e^+ + e^- \rightarrow q + \bar{q}$, leading to the production of a quark-antiquark pair; followed by *fragmentation*:



Jet angular distribution relative to the electron beam direction reflects the angular distributions of quark and antiquark in the basic reaction. This is another piece of evidence for the existence of quarks. Occasionally a high momentum gluon is emitted by the quark or anti-quark before fragmentation occurs, (cf bremssrahlung). Quark, antiquark and gluon fragment into hadrons, leading to a *three-jet* event. Provided the first unambiguous evidence for gluons, since the relative angular distributions of the jets are in good agreement with the theoretical expectation for spin-1 gluons. Ratio of three-jet to two-jet events determined by α_s and $\Rightarrow$ value for Λ that is consistent with other determinations.

Colour counting

Consider ratio

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

Suppose each quark flavour $f = u, d, s, ...$ exists in N_C colour states ($N_C = 3$ according to QCD, $N_C = 1$ if the colour degree of freedom does not exist). Since colour states all have the same electric charge, all are produced equally readily by two-step mechanism $\Rightarrow$ rate for producing quark pairs of any given flavour $f = u, d, s, ...$ will be proportional to number of colours N_C. Cross-section is also proportional to squared charge of the produced pair, and since muon pairs are produced by an identical mechanism, we obtain

$$\sigma(e^+e^- \rightarrow q\bar{q}) = N_C\, e_f^{\,2} \sigma(e^+e^- \rightarrow \mu^+\mu^-)$$

where e_f is the electric charge on a quark of flavour *f*. Hence if hadron production were completely dominated by the two-step process $\Rightarrow$

$$R = R_0 \equiv N_C(e_u^{\,2} + e_d^{\,2} + e_s^{\,2} + e_c^{\,2} + e_b^{\,2}) = 11N_C/9$$

since the top quark is too heavy to be produced at these energies.

Including small contribution from the three-jet events and other corrections of order α_s:

$$R = R_0(1+\alpha_s/\pi)$$

giving rise to a weak energy dependence from the running of α_s described above. Data are consistent with this for $N_C = 3$ (see Fig.5.11).

Deep inelastic scattering: nucleon structure

The inelastic scattering of a charged lepton from a proton is shown in Fig.5.12. Described by two independent variables: ν, defined by

$$2M\nu \equiv W^2c^2 + Q^2 - M^2c^2$$

and dimensionless quantity (*scaling variable*) $x \equiv Q^2/2M\nu$ (fraction of momentum of target proton carried by the quark which interacts with, i.e. is struck by, exchanged photon), M = proton mass, W = invariant mass of final-state hadrons, Q^2 = squared energy-momentum transfer $Q^2 = (\mathbf{p}-\mathbf{p}')^2 - (E-E')^2/c^2$. Need also to introduce two form factors W_1 and W_2, (*structure functions*). Then:

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2(\theta/2)\right]$$

θ = lepton scattering angle. Define:

$$F_1(x,Q^2) \equiv Mc^2W_1(Q^2,\nu) \quad \text{and} \quad F_2(x,Q^2) \equiv \nu W_2(Q^2,\nu)$$

For $W \geq 2.5\ \mathrm{GeV}/c^2$, structure functions (see Fig 5.13) have only a very weak dependence on Q^2 at fixed values of $x \Rightarrow$ proton has sub-structure of point-like charge constituents. Other experiments (using neutrino beams as well as neutron targets) enable their spins and charges to be deduced and the identification with quarks to be confirmed.

Scaling, i.e. independence of the data on Q^2 at fixed x, is not exact (see Fig.5.15) Deviations are due to QCD corrections to simple quark model, i.e. the quark in the proton that is struck by exchanged photon can itself radiate gluons. Scaling violations are explained by QCD using a value of Λ consistent with that obtained from other souces.