3. LEPTONS, QUARKS AND HADRONS

Lepton multiplets
Three *generations*:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

Each has a *charged lepton* with electric charge -*e*, and a neutral *neutrino*. Charged leptons (e^-, μ^-, τ^-): electron, *mu lepton* (*muon*, or just *mu*) and *tau lepton* (*tauon*, or just *tau*). Associated neutrinos: *electron neutrino*, *mu neutrino*, and *tau neutrino*. Antileptons are:

$$\begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}, \quad \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}, \quad \begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix}$$

Charged leptons interact via both electromagnetic and weak forces; for neutrinos, only weak interactions have been observed. The masses and lifetimes of the leptons are listed in a Table in the lectures. Electron and the neutrinos are stable; muons decay by the weak interactions

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \;\; ; \quad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

The tauon also decays by the weak interaction, but with a much shorter lifetime. Because it is heavier than the muon, it can decay to many different final states, which can include both hadrons and leptons. 35% of decays lead to purely leptonic final states, *via* reactions that are very similar to muon decay, for example:

$$\tau^+ \to \mu^+ + \nu_\mu + \bar{\nu}_\tau \;\; ; \quad \tau^- \to e^- + \bar{\nu}_e + \nu_\tau$$

These decays illustrate the principle of *lepton number conservation*.

Lepton numbers
Associated with each generation of leptons is a *lepton number*, e.g. *electron number*:

$$L_e \equiv N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e)$$

$N(e^-)$ = the number of electrons present, $N(e^+)$ = the number of positrons present etc. $L_e = 1$ for e^- and ν_e; $L_e = -1$ for e^+ and $\bar{\nu}_e$; and $L_e = 0$ for all other particles. *muon* and *tauon numbers* defined in a similar way. For multiparticle states the lepton numbers of the individual particles are added. Each lepton number is conserved in *any* reaction; reactions that violate lepton number conservation are "forbidden". Explains stability of electron and neutrinos. The electron is stable because electric charge is conserved in all interactions and the electron is the lightest charged particle. Hence decays to lighter particles which satisfy all other conservation laws, like $e^- \to \nu_e + \gamma$, are forbidden by electric charge conservation. In the same way, lepton number conservation implies that the lightest particles with non-zero values of the three lepton numbers – the three neutrinos – are stable, whether they have zero masses or not.

Neutrinos
Existence of ν_e postulated to understand β-decays. The neutrinos and antineutrinos emitted in these decays are not observed experimentally, but are inferred from energy and angular momentum conservation. In the case of energy, if the antineutrino were not present, the energy E_e of the emitted electron would be equal to the difference in rest energies of the two nuclei

$$E_e = \Delta M c^2 = \left[M(Z,N) - M(Z+1,N-1) \right] c^2$$

If the antineutrino is present, the electron energy would not be unique, but would lie in the range

$$m_e c^2 \le E_e \le (\Delta M - m_{\bar{\nu}_e}) c^2$$

Experimentally, the observed energies span the whole of the above range $\Rightarrow$ least one additional particle is present in final state. Mass of neutrinos are small (experimental limits are given in Table in lectures).

Neutrinos can only be detected with extreme difficulty because they only interact via the weak interaction; e.g. neutrinos and antineutrinos emitted in β-decays, with energies of order 1MeV, have mean free paths in matter of order 10^6 km. Electron electron neutrinos have been detected by inverse β–decay using flux from reactors. ν_μ has been detected using the reaction $\nu_\mu + n \to \mu^- + p$ and other reactions. In 2000, a few examples of tau neutrinos were reported, so all three types have now been directly detected.

Neutrino mixing and oscillations
Neutrinos have zero masses in the standard model. However, from the β-decay of tritium there is evidence for a non-zero mass. $\Rightarrow$ *neutrino mixing*. Arises if we assume that the observed neutrino states ν_e, ν_μ and ν_τ that take part in weak interactions are not eigenstates of mass, but instead are linear combinations of three other states ν_1, ν_2 and ν_3 that do have definite masses m_1, m_2 and m_3. Consider the case of mixing between just two states:

$$\nu_e = \nu_1 \cos\alpha + \nu_2 \sin\alpha$$

and

$$\nu_\mu = -\nu_1 \sin\alpha + \nu_2 \cos\alpha$$

α is a *mixing angle* to be determined from experiment. Measurement of α may be done by studying the phenomenon of *neutrino oscillation*. This effect follows from simple quantum mechanics. The formalism is given in detail in the lectures. The probability of finding a muon neutrino in a beam of initially pure electron neutrinos is

$$P(\nu_e \to \nu_\mu) = |B(t)|^2 = \sin^2(2\alpha) \sin^2\left[(E_2 - E_1) t / 2\hbar \right]$$

where $E_{1,2}$ are the energies of states 1 and 2. This oscillates with time, while the probability of finding an electron neutrino is reduced by a corresponding oscillating factor. Similar effects are predicted if instead we start from muon neutrinos. In both cases the oscillations vanish if the mixing angle is zero, or if the neutrinos have equal masses, and hence equal energies. Clear evidence for the existence of neutrino oscillations was obtained from observations on *atmospheric neutrinos* by the giant *Super Kamiokande* detector. When cosmic ray protons collide with atoms in the upper atmosphere, they create many pions, which in turn create neutrinos mainly by the decay sequences

$$\pi^- \to \mu^- + \bar{\nu}_\mu\,, \qquad \pi^+ \to \mu^+ + \nu_\mu$$

and

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu\,, \qquad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

From this, one would naively expect to see two muon neutrinos for every electron neutrino detected. However, the ratio was observed to be about 1.3 to 1 on average, suggesting that the muon neutrinos produced might be oscillating into other species. Confirmed by studying azimuthal dependence of the effect: compare measured flux from neutrinos produced in the atmosphere directly above the detector, which have a short flight path before detection, with those incident from directly below, which have traveled a long way through the earth before detection, and so have had plenty of time to oscillate. Yield of electron neutrinos from above and below were the same within errors and consistent with expectation for no oscillations. Yield of muon neutrinos from above accorded with the expectation for no significant oscillations, but flux of muon neutrinos from below was a factor of about two lower $\Rightarrow$ muon neutrino oscillations, presumably into tauon neutrinos.

The existence of neutrino oscillations, and by implication non-zero neutrino masses, is now generally accepted on the basis of the above and other evidence. However the details, including the values of the neutrino mass differences and the various mixing angles involved, remain to be resolved and this will be done in a number of experiments that will detect oscillations directly using prepared neutrino beam and making measurements at great distances from their origin. One such experiment is called MINOS and members of our own particle physics group are involved in this, which aims to measure these parameters in the next few years.

What are the consequences of these results for the standard model? The observation of oscillations does not lead to measurements of neutrino masses, only (squared) mass differences, but combined with the tritium experiment it would be natural to assume that neutrinos all had very small masses, with the mass differences being of the same order-of-magnitude as the masses themselves. The standard model can be modified to accommodate such small masses, although methods for doing this is are not without their own problems. (See Sec 6 for one way of doing this.) Also consequences for lepton number conservation. In the simple mixing model above, total lepton number could still be conserved, but individual lepton numbers would not. However, there are other theoretical descriptions of neutrino oscillations and this is an open question. A definitive answer would be to detect *neutrinoless double β-decay*, such as

$$^{76}Ge \to {}^{76}Se + 2e^-$$

as the final state contains two electrons, but no antineutrinos. A very recent experiment claims to have detected this decay, but the result is not universally accepted and at present 'the jury is still out'. Experiments planned for the next few years should settle this very important question.

Universal lepton interactions; numbers of neutrinos

All three neutrinos have similar properties, but the three charged leptons are strikingly different. Examples: magnetic moment of the electron is ~ 200 times greater than that of the muon; high energy electrons are mostly stopped by 1 cm of lead, while muons are the most penetrating form of radiation known, apart from neutrinos etc. Nevertheless, all experimental

data are consistent with the assumption that the interactions of each generation are identical, *provided the mass differences are taken into account.* Called *universality* and is verified with great precision (examples are given in the lectures).

Are there more generations of neutrinos? Can be answered from decays of the Z^0 boson. It decays, among other final states, to neutrino pairs

$$Z^0 \to \nu_\ell + \bar{\nu}_\ell \qquad (\ell = e, \mu, \tau)$$

with a total rate proportional to the number of neutrino species, if we assume universal lepton interactions and neutrino masses which are small compared to the mass of the Z^0 (such that Z^0 can decay to them). Total decay rate is

$$\Gamma_{total} = \Gamma_{hadrons} + \Gamma_{charged\ leptons} + \Gamma_{neutrinos}$$

The first three rates can be measured and hence the rate to neutrinos deduced. Comparison with the theoretical prediction (N_ν times rate to each specific $\nu\bar{\nu}$ pair) gives $N_\nu = 3$ with a small error $\Rightarrow$ only three generations of leptons can exist.

Evidence for quarks

Quarks never directly observed as single, free particles. Free quarks should be easy to detect (ionization etc); failure is not an experimental problem, but an intrinsic property of quarks (*confinement*). Evidence is indirect: from *hadron spectroscopy, lepton scattering* and *jets.*

Hadron spectroscopy. Study of the static properties of hadrons. Close correspondence between the experimentally observed hadrons and those predicted by the quark model.
Lepton scattering. Rutherford-type experiments performed with high-energy leptons scattered from protons and neutrons yield evidence for point-like entities within the nucleons (quarks).
Jets. High-energy collisions cause the quarks within hadrons, or newly created quark-antiquark pairs, to fly apart from each other with very high energies. Before they can be observed, these quarks are converted (*fragmentation*) into jets of hadrons, whose production rates and angular distributions reflect those of the quarks from which they originated.

Properties of quarks

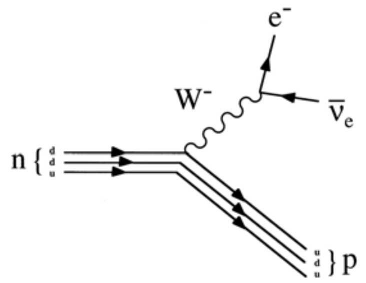
Six *flavours*, occurring in three *generations*:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Each generation: quark with charge +2/3 (u, c, or t) and quark with charge –1/3 (d, s, or b), in units of e. Called *down* (d), *up* (u), *strange* (s), *charmed* (c), *bottom* (b) and *top* (t) quarks. Quantum numbers associated with the s, c, b and t quarks: *strangeness, charm, beauty* and *truth*, respectively. Antiquarks:

$$\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}, \quad \begin{pmatrix} \bar{s} \\ \bar{c} \end{pmatrix}, \quad \begin{pmatrix} \bar{b} \\ \bar{t} \end{pmatrix}$$

Approximate quark masses are given in a Table in lectures. Except for the top quark, masses are inferred indirectly from the observed masses of their hadron bound states, together with models of quark binding. *s, c* and *b* quarks can be assigned approximate lifetimes of $10^{-8}-10^{-10}$ s for the *s*-quark and $10^{-12}-10^{-13}$ s for both the *c*- and *b*-quarks. The top quark is much heavier than the other quarks and its lifetime is of order 10^{-25} s. This lifetime is so short that top quarks decay too quickly to form observable hadrons. "The decay of quarks" always means that the decay takes place within a hadron, with the other bound quarks acting as "spectators". For example, neutron decay in this picture is given by the Feynman-like *quark diagram*:



Quark numbers
Defined by:

$$N_f \equiv N(f) - N(\bar{f}) \qquad (f = u, d, s, c, b, t)$$

$N(f)$ = number of quarks of flavour *f* present; $N(\bar{f})$ = the number of $\bar{f}$-antiquarks present. For single-particle states; $N_c = 1$ for the *c*-quark; $N_c = -1$ for the $\bar{c}$ antiquark; and $N_c = 0$ for all other particles. For multi-particle states the quark numbers of the individual particles are simply added. In weak interactions, only the total quark number

$$N_q \equiv N(q) - N(\bar{q})$$

is conserved, where $N(q)$ and $N(\bar{q})$ are the total number of quarks and antiquarks present, irrespective of their flavour. Example: the *main* decay mode of the charmed quark:

$$c \to s + u + \bar{d}$$

clearly violates conservation of the quark numbers N_c, N_s, N_u and N_d, but the total quark number N_q is conserved. In practice, it is convenient to replace the total quark number N_q by the *baryon number,*

$$B \equiv N_q/3 = [N(q) - N(\bar{q})]/3 \ .$$

Baryon number is conserved in *all known interactions.*

Hadrons
Three types observed: *baryons* (half-integer spin, 3*q*); *antibaryons* (half-integer spin, $3\bar{q}$); *mesons* (integer spin, $q\bar{q}$). These assumptions constitute the *quark model of hadrons.* The lightest known baryons are the proton and neutron ($p = uud$, $n = udd$). Pions are the lightest known mesons and have the quark compositions

$$\pi^+ = u\bar{d}\ , \quad \pi^0 = u\bar{u}, d\bar{d}\ , \quad \pi^- = d\bar{u}$$

Kaons are the lightest strange mesons, with the quark compositions:

$$K^+(494) = u\bar{s}\ , \qquad K^0(498) = d\bar{s}$$

where K^+ and K^0 have *S* = +1 and K^- and $\bar{K}^0$ have *S* = –1, while the lightest strange baryon is the *lambda,* with the quark composition $\Lambda = uds$. Hadrons containing *c* and *b* quarks have also been discovered, with non-zero values of the *charm* and *beauty* quantum numbers defined by

$$C \equiv N_c \equiv N(c) - N(\bar{c}) \ \text{ and } \ \tilde{B} \equiv -N_b \equiv -N(b) - N(\bar{b})$$

(Note the minus sign in *C,* just as in the definition of *S.*)

To proceed systematically one would (e.g.) construct all the mesons states of the form $q\bar{q}$ where *q* can be any of the six quark flavours. The simplest such states would have the spins of the two quarks antiparallel with no orbital angular momentum between them, and so have spin-0. If, for simplicity, we such consider those states composed of *u, d* and *s* quarks, you can easily find that the nine bosons have quantum numbers which may be identified with the observed mesons (K^0, K^+), $(\bar{K}^0, K^-)$, $(\pi^\pm, \pi^0)$ and two neutral particles, which are called η and η'. This can be extended to the lowest lying baryon states *qqq* and also to all six quark flavours. Fact: the states observed experimentally agree with those predicted by the simple combinations $qqq, \bar{q}\bar{q}\bar{q}$ and $q\bar{q}$, and there is no convincing evidence for states corresponding to other combinations.

In the quark model excited states (*resonances)* can be formed with non-zero orbital angular momentum between the quarks. The quark model successfully accounts for the excited states of the various quark systems, as well as their ground states. Resonances usually decay via strong interaction. Example: $K^{*+}(890) = u\bar{s}$ resonance, which decays to $K^+\pi^0$ and $K^0\pi^+$ final states with a lifetime of 1.3x10^{-23} s. Quark description of the process $K^{*+} \to K^0 + \pi^+$ (e.g) is $u\bar{s} \to d\bar{s} + u\bar{d}$ (conserves the quark numbers N_u and N_d are separately.) This is characteristic of strong and electromagnetic processes, which are only allowed if all the quark numbers N_u, N_d, N_s, N_c , and N_b are separately conserved. Hadrons that do not decay via the strong or electromagnetic interactions can only decay by the weak interaction, which can violate quark number conservation. Examples: π^+ decay (lifetime 2.6x10^{-8} s): $\pi^+ \to \mu^+ + \nu_\mu$; $\Lambda(1116) = uds$ decay (lifetime 2.6x10^{-10} s): $\Lambda \to p + \pi^-$etc. The quark interpretations are $(u\bar{d}) \to \mu^+ + \nu_\mu$, in which a *u*-quark annihilates with a $\bar{d}$-antiquark, violating both N_u and N_d conservation; and for lambda decay $sud \to uud + d\bar{u}$ in which an *s* quark turns into a *u* quark and a $u\bar{d}$ pair is created, violating N_d and N_s conservation. Lifetimes are summarized in a Table. In the lectures.

Flavour independence and charge multiplets
Flavour independence: the strong force between two quarks at a fixed distance apart is independent of which quark flavours u, d, s, c, b, t are involved. The same principle applies to quark-antiquark forces, but these are *not* identical to quark-quark forces. Flavour independence does not apply to the electromagnetic interaction, since the quarks have different electric charges. In applying flavour independence one must take account of the

quark mass differences. Consequences: hadrons occur in families of particles with approximately the same masses, called *charge multiplets.* electric charges. Examples: triplet of pions, (π^+, π^0, π^-), and the nucleon doublet (p, n). Symmetry is not exact because of the small mass difference between the u and d quarks and because of the electromagnetic forces, and it is these that lead to the small differences in mass within multiplets. The symmetry between u and d quarks is called *isospin symmetry* and greatly simplifies the interpretation of hadron physics. Flavour independence of the strong forces between u and d quarks also leads directly to the *charge independence of nuclear forces*. A case where the mass differences between quarks are large, but relatively easily taken into account, is the comparison of the $c\bar{c}$ and $b\bar{b}$ quark systems (*charmonium* and *bottomium*). The quarks are so heavy that they move slowly enough within the resulting hadrons to be treated non-relativistically to a first approximation. The rest energies of the bound states, and hence their masses, can be calculated from the static potential between the quarks (cf the calculation of the energy levels of hydrogen atom from the Coulomb potential). In this case the procedure is reversed, with the aim of determining the form of the static potential from the rather precisely measured energies of the bound states. To cut a long story short, one finds that the potentials required to describe the system are the same within the reasonably small uncertainties of the method, confirming again the flavour independence of the strong force.