2. NUCLEAR PHENOMENOLOGY

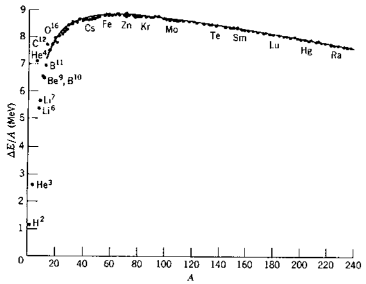
Notation
Nuclei are specified by ${}^{A}X$ or ${}^{A}_{Z}X$, where X is the chemical symbol for the element:

Z – Atomic Number = number of protons (charge on nucleus is $+Ze$)
N – Neutron Number = number of neutrons
A – Mass Number = number of nucleons
Nuclei with the same mass number (*isobars*); same atomic number (*isotopes*)

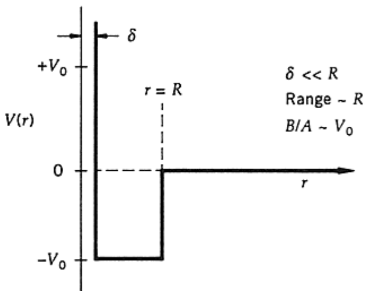
Masses and binding energies
The mass deficit

$$\Delta M(Z,A) = M(Z,A) - Z\,M_p - N\,M_n$$

$-\Delta Mc^2$ is the *binding energy B*. The *binding energy per nucleon* B/A, curve is



Nuclear (nucleon-nucleon) forces
Overall *attractive* and much stronger than the Coulomb force; repulsive core at very short ranges; *short-range*, of the same order as the size of the nucleus (nucleon-nucleon scattering experiments) Schematically,



Nuclear force is *charge-symmetric* ($pp = nn$) (comparison of *pp* and *nn* scattering); almost *charge-independent* ($pp = nn = pn$) (mirror nuclei); strongly *spin-dependent* (force between a proton and neutron in an overall spin-1 state supports a bound state – *deuteron* – whereas the potential corresponding to the spin-0 state has no bound state); nuclear forces *saturate* – nucleons in nucleus experience attractive interactions only with a limited number of the many other nucleons (B/A largely independent of *A*).

Shapes and sizes
Found from scattering experiments. Basic formula for electron scattering:

$$\left(\frac{d\sigma}{d\Omega}\right)_{expt} = \frac{Z^2\alpha^2(\hbar c)^2}{4E^2\sin^4(\theta/2)}\left[1-\beta^2\sin^2(\theta/2)\right]\left|F(q)\right|^2$$

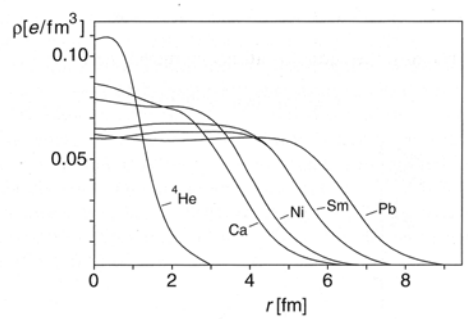
$\beta = v/c$, v is the velocity of the initial electron, *form factor* $F(\mathbf{q}^2)$ is defined in terms of the spatial charge distribution $f(\mathbf{x})$ by

$$F(\mathbf{q}) \equiv \int e^{i\mathbf{q}.\mathbf{x}/\hbar} f(\mathbf{x})\, d^3\mathbf{x}$$

q is the momentum transfer for the electron. Measurements of the cross-section for a fixed energy at various angles (and hence various **q**), yield form factor from the inverse Fourier transform

$$f(\mathbf{x}) = \frac{1}{(2\pi)^3}\int F(\mathbf{q}) e^{-i\mathbf{q}.\mathbf{x}/\hbar}\, d^3\mathbf{q}$$

In practice, the differential cross-section decreases extremely rapidly with angle and cannot be measured over a sufficiently wide range of angles for the integral to be evaluated accurately. Instead, a parameterised form is chosen for $f(\mathbf{x})$, then the form factor is calculated from the Fourier transform and a fit made to the data using the resulting expression for the differential cross-section. Examples:



Fitted by

$$\rho_{ch}(r) = \frac{\rho_{ch}{}^{0}}{1+e^{(r-c)/a}}$$

⇒ charge density is approximately constant in the nuclear interior and falls fairly rapidly to zero at the nuclear surface. Nucleus often approximated by a homogeneous charged sphere of radius *R* (*nuclear radius*) given by

$$R_{charge} = 1.21 A^{1/3}\,\mathrm{fm}$$

Nuclear (i.e. matter) density of nuclei probed using *hadron* projectiles. Results very similar to charge distributions. Interior nuclear density is $\rho_{nucl} \approx 0.17$ nucleons / fm^3; effective nuclear matter radius for medium and heavy nuclei is

$$R_{nuclear} \approx 1.2 A^{1/3}\ fm$$

Liquid drop model: semi-empirical mass formula (SEMF)
Nucleus assumed to be an incompressible liquid droplet where the nucleons play the role of individual molecules within the droplet $\Rightarrow$ *semi-empirical mass formula* (plays an important role in the discussion of nuclear stability). Uses: (1) interior mass densities of nuclei approximately equal; (2) total binding energies approximately proportional to their masses. SEMF is

$$M(Z,A) = \sum_{i=0}^{5} f_i(Z,A)$$

First terms are the *mass of the constituent nucleons*,

$$f_0(Z,A) = Z M_p + (A-Z) M_n$$

The remaining terms are various corrections, written in the form a_i multiplied by a function of Z and A with $a_i > 0$.
(a) volume term,

$$f_1(Z,A) = -a_1 A$$

arises from saturation of nuclear force $\Rightarrow$ binding energy proportional to volume, or nuclear mass (nuclear radius proportional to $A^{1/3}$); negative coefficient; i.e. increases binding energy.

(b) surface term (correction to volume term because nucleons at the surface are not surrounded by other nucleons)

$$f_2(Z,A) = +a_2 A^{2/3}$$

(c) Coulomb term (protons repel each other)

$$f_3(Z,A) = +a_3 \frac{Z^2}{A^{1/3}}$$

(d) asymmetry term,

$$f_4(Z,A) = +a_4 \frac{(Z-A/2)^2}{A}$$

(tendency for nuclei to have $Z = N$). This form can be 'checked' by considering the filling of topmost energy levels – due to Pauli principle.

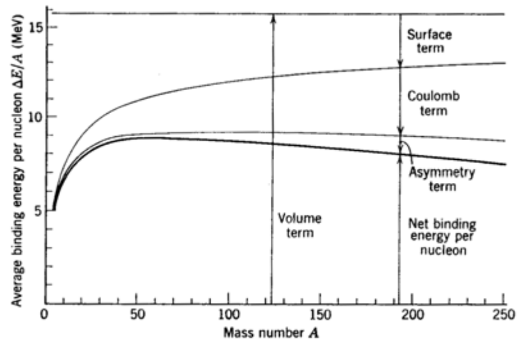
(e) pairing term (empirical) – maximises the binding when both Z and N are even.

$f_5(Z,A) = -f(A)$, if Z even, $A-Z=N$ even
$f_5(Z,A) = 0$, if Z even, $A-Z=N$ odd; or, Z odd, $A-Z=N$ even
$f_5(Z,A) = +f(A)$, if Z odd, $A-Z=N$ odd

$f(A) = a_5 A^{-1/2}$ is obtained by fitting data. Useful notation:

$$a_1 = a_v,\quad a_2 = a_s,\quad a_3 = a_c,\quad a_4 = a_a,\quad a_5 = a_p$$

Relative sizes of each of the terms:



Nuclear stability
Stable nuclei only occur in a very narrow band in the $Z-N$ plane All other nuclei are unstable and decay spontaneously in various ways.

β – decay : phenomenology
SEMF may be written as a quadratic in Z at fixed A:

$$M(Z,A) = \alpha A - \beta Z + \gamma Z^2 + \frac{\delta}{A^{1/2}}$$

SEMF now applied to *atomic* masses. For odd A, the curve is a single parabola. For even A, two curves because of the pairing term. The minimum of the parabolas is at $Z = \beta/2\gamma$. The nucleus with the smallest mass in an isobaric spectrum is stable with respect to β-decay.

(a) Odd-mass nuclei
Electron emission, $n \to p + e^- + \bar{\nu}_e$, is energetically possible whenever the mass of the daughter atom $M(Z+1,A)$ is smaller than its isobaric neighbour, i.e.

$$M(Z,A) > M(Z+1,A)$$

Isobars with proton excess decay via *positron emission*, $p \to n + e^+ + \nu_e$ (not possible for a free proton, but allowed in a nucleus because of the binding energy). Energetically possible if

$$M(Z,A) > M(Z-1,A) + 2m_e$$

which takes account of the creation of a positron and the existence of an excess of electrons in the parent atom. *Electron capture*, $e^- + p \to n + \nu_e$, also possible if

$$M(Z,A) > M(Z-1,A) + \varepsilon$$

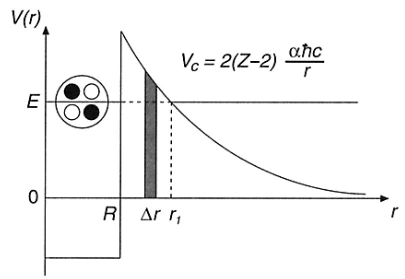
– ε is the excitation energy of the atomic shell of the daughter nucleus – competes with positron emission (occurs in heavy nuclei, where the electron orbits are more compact).

(b) Even-mass nuclei
Mass conditions are the same – two curves, with the odd-odd curve lying above even-even curve. The lowest isobar on the lowest curve is β-stable. It is also a common situation to

have another isobar on the lower curve with its two odd-odd neighbours above it. Odd-odd nuclei always have at least one more strongly bound, even-even neighbour nucleus in the isobaric spectrum $\Rightarrow$ therefore unstable. Only exceptions are a few very light nuclei. Lifetimes of β emitters vary enormously from milliseconds to 10^{16} yrs. They depend very sensitively on the energy released and on the properties of the nuclei involved, e.g. their spins.

α-decay

SEMF can be used to show that above about $A = 151$ α-decay becomes energetically possible. Lifetimes of α-emitters also span an enormous range – from 10 ns to 10^{17} yrs. α-decay is a tunelling phenomenom. Individual protons and neutrons have binding energies in nuclei of about 8 MeV, even in heavy nuclei, and so cannot in general escape, but a bound group of nucleons can sometimes escape because its binding energy increases the total energy available for the process. Most significant decay process of this type is the emission of an α-particle, which is very strongly bound by $7\,\mathrm{MeV/nucleon}$. Potential energy of an α-particle as a function of r, its distance from the centre of the nucleus:



Beyond the nuclear force range $r > R$, the α-particle feels only the Coulomb potential

$$V_C(r) = \frac{2(Z-2)\alpha\hbar c}{r}$$

Within the nuclear force range $r < R$, a strong nuclear potential prevails. The α-particle has $E_\alpha > 0$, but less than the Coulomb barrier $\Rightarrow$ α-particle must quantum mechanically tunnel through the barrier. The probability per unit time λ of the α-particle escaping is proportional to: (a) the probability $w(\alpha)$ of finding the α-particle in the nucleus; (b) the number of collisions of the α-particle with the barrier (this is proportional to $v_0/2R$ where v_0 is the velocity of the α-particle within the nucleus); and (c) the transition probability. Thus

$$\lambda = w(\alpha)\frac{v_0}{2R}e^{-2G}$$

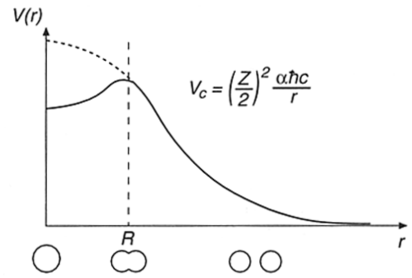
where the *Gamow factor G* is

$$G = \frac{1}{\hbar}\int_R^{r_1}\sqrt{2m\left|E_\alpha - V(r)\right|}\,dr \approx \frac{2\pi\alpha(Z-2)}{\beta}$$

Small differences in E_α have strong effects on the lifetime.

Fission

Spontaneous fission – two daughter nuclei are of approximately equal mass. Probability increases with increasing *A*, but still a very rare process. SEMF assumes the nucleus is spherical (minimises surface area). If the surface is perturbed to prolate, the surface term increases and the Coulomb term decreases (assuming constant volume). Relative sizes of changes determines whether the nucleus is stable against spontaneous fission. $\Delta E < 0$ $\Rightarrow$ deformation energetically favourable (fission occurs). This happens for nuclei with $Z > 114$ and $A \geq 270$. Spontaneous fission is a potential barrier problem:



Another possibility is to supply this energy by a flow of neutrons. A neutron can get very close to the nucleus and be captured by the strong nuclear attraction. The parent nucleus may then be excited to a state above the fission barrier and therefore split up – called *induced fission*. Neutron capture by a nucleus with an odd neutron number releases binding energy *and* pairing energy. This small extra contribution makes a crucial difference to nuclear fission properties. Very low-energy ('thermal') neutrons can induce fission in ^{235}U, whereas only higher energy ('fast') neutrons induce fission in ^{238}U. This is because ^{235}U is an even-odd nucleus and ^{238}U is even-even. Therefore, the ground state of ^{235}U will lie higher (less tightly bound) in the potential well of its fragments than that of ^{238}U. Hence to induce fission, a smaller energy will be needed for ^{235}U than for ^{238}U.

Qualitatively: the capture of a neutron by ^{235}U changes an even-odd nucleus to a more tightly bound even-even (compound) nucleus of ^{236}U and releases the binding energy of the last neutron. In ^{235}U this is 6.5 MeV. The energy needed to induce fission (i.e. the *activation energy*) is calculated to be about 5 MeV for ^{236}U and thus neutron capture releases sufficient energy to fission the nucleus. The kinetic energy of the incident neutron is irrelevant and thermal neutrons can induce fission in ^{235}U. In contrast, neutron capture in ^{238}U changes it from an even-even nucleus to an even-odd nucleus, i.e. changes a tightly bound nucleus to a less tightly bound one. The energy released (the binding energy of the last neutron) is about 4.8 MeV in ^{239}U and is less than the 6.5 MeV required for fission. Thus fast neutrons with an energy of at least this difference are required.

γ – decays

Disintegratation by α or β decay, or by fission, often leaves daughter nucleus in an excited state. If this state does not itself also disintegrate, it will de-excite, usually by emitting a high-energy photon. The energy of these photons is a few to several MeV. Because γ-decay is an electromagnetic process, typical lifetimes $\sim 10^{-16}$ s. In practice, lifetimes are very sensitive to the amount of energy released and other factors are also very important, particularly the quantity of angular momentum carried off by the photon. Typical lifetimes of nuclear levels are about $\sim 10^{-12}$ s. The role of angular momentum in γ-decays is crucial.