3. LEPTONS, QUARKS AND HADRONS

We turn now to some of the basic phenomena of particle physics: the properties of leptons and quarks, and the bound states of the latter, the hadrons.

3.1 Lepton multiplets

We have seen that the spin-1/2 leptons are one of the three classes of elementary particles in the standard model. There are six known leptons, and they occur in pairs, each called a *generation*, which we write, for reasons that will become clear presently, as:

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}, \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}, \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}$$

Each generation comprises a *charged lepton* with electric charge $-e$, and a neutral *neutrino*. The three charged leptons (e^-, μ^-, τ^-) are the familiar electron, together with two heavier particles, the *mu lepton* (usually called the *muon*, or just *mu*) and the *tau lepton* (usually called the *tauon*, or just *tau*). The associated neutrinos are called the *electron neutrino*, *mu neutrino*, and *tau neutrino*, respectively. In addition to the leptons there are six corresponding antileptons:

$$\begin{pmatrix}e^+\\ \bar{\nu}_e\end{pmatrix}, \quad \begin{pmatrix}\mu^+\\ \bar{\nu}_\mu\end{pmatrix}, \quad \begin{pmatrix}\tau^+\\ \bar{\nu}_\tau\end{pmatrix}$$

Ignoring gravity, the charged leptons interact via electromagnetic and weak forces, whereas for the neutrinos, only weak interactions have been observed. Because of this, neutrinos, which are all believed to have extremely small masses, can be detected only with considerable difficulty.

The masses and lifetimes of the leptons are listed for convenience in Table 3.1.

Table 3.1 Properties of leptons. All have spin-1/2. Masses are given units of MeV/c^2. The antiparticles (not shown) have the same masses as their associated particles, but the electric charges (Q) and lepton numbers (L_ℓ, $\ell = e, \mu, \tau$) are reversed in sign.

Name and symbol	Mass	Q	L_e	L_μ	L_τ	Lifetime (s)	Major decays
Electron e^-	0.511	–1	1	0	0	Stable	None
Electron neutrino ν_e	$< 3\mathrm{eV}/c^2$	0	1	0	0	Stable	None
Muon (mu) μ^-	105.7	–1	0	1	0	$2.197\text{x}10^{-6}$	$e^-\bar{\nu}_e\nu_\mu$ (100%)
Muon neutrino ν_μ	< 0.19	0	0	1	0	Stable	None
Tauon (tau) τ^-	1777.0	–1	0	0	1	$2.91\text{x}10^{-13}$	$\mu^-\bar{\nu}_\mu\nu_\tau$ (17.4%) $e^-\bar{\nu}_e\nu_\tau$ (17.8%) ν_τ+hadrons (~64%)
Tauon neutrino ν_τ	< 18.2	0	0	0	1	Stable	None

The electron and the neutrinos are stable, for reasons that will become clear shortly. The muons decay by the weak interaction processes

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \ ; \quad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

with lifetimes $2.2\text{x}10^{-6}\,\text{s}$. The tauon also decays by the weak interaction, but with a much shorter lifetime $(2.91 \pm 0.02)\text{x}10^{-13}\,\text{s}$. Because it is heavier than the muon, the tauon decays to many different final states, which can include both hadrons and leptons. However about 35% of decays again lead to purely leptonic final states, *via* reactions which are very similar to muon decay, for example:

$$\tau^+ \to \mu^+ + \nu_\mu + \bar{\nu}_\tau \ ; \quad \tau^- \to e^- + \bar{\nu}_e + \nu_\tau$$

These decays illustrate the fundamental principle of *lepton number conservation*.

3.2 Lepton numbers

Associated with each generation of leptons is a quantum number called a *lepton number*. The first of these *lepton numbers* is the *electron number*, defined for any state by

$$L_e \equiv N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e)$$

where $N(e^-)$ is the number of electrons present, $N(e^+)$ is the number of positrons present and so on. For single-particle states, $L_e = 1$ for e^- and ν_e; $L_e = -1$ for e^+ and $\bar{\nu}_e$; and $L_e = 0$ for all other particles. The *muon* and *tauon numbers* are defined in a similar way and their values for all single particle states are summarized in Table 3.1. They are zero for all particles other than leptons, such as photons, protons or neutrons. For multiparticle states the lepton numbers of the individual particles are simply added. For example, the final state in neutron β-decay (i.e. $n \to pe^-\bar{\nu}_e$) has

$$L_e = L_e(p) + L_e(e^-) + L_e(\bar{\nu}_e) = (0) + (1) + (-1) = 0$$

like the initial state, which has $L_e(n) = 0$.

In the standard model, the value of each lepton number is postulated to be conserved in *any* reaction. [However, see Sec 3.4 for recent evidence for lepton number non-conservation.] In electromagnetic interactions, this reduces to the conservation of $N(e^-) - N(e^+)$, $N(\mu^-) - N(\mu^+)$, and $N(\tau^-) - N(\tau^+)$, since neutrinos are not involved. This implies that the charged leptons can only be created or annihilated in particle-antiparticle pairs.

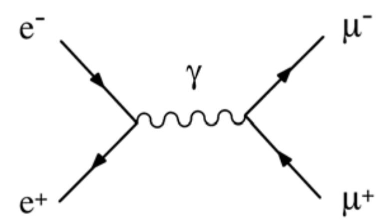

Fig.3.1 Single-photon exchange in the reaction $e^+e^- \to \mu^+\mu^-$

For example, in the electromagnetic reaction

$$e^+ + e^- \to \mu^+ + \mu^-$$

an electron pair is annihilated and a muon pair is created by the mechanism of Fig.3.1. In weak interactions more general possibilities are allowed, which still conserve the lepton numbers. For example, in the tau-decay process $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$, a tauon converts to a tauon neutrino and an electron is created together with an antineutrino, rather than a positron. The dominant Feynman graph corresponding to this process is shown in Fig.3.2.

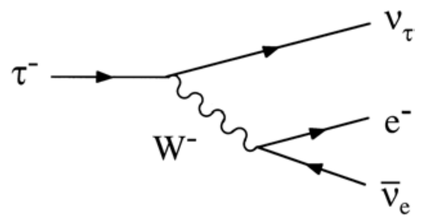

Fig.3.2 Dominant Feynman diagram for the decay $\tau^- \to e^- \bar{\nu}_e \nu_\tau$

Lepton number conservation, like electric charge conservation, plays an important role in understanding reactions involving leptons. Observed reactions conserve lepton numbers, while reactions which violate lepton number conservation are "forbidden" and are not observed. For example, the neutrino scattering reaction

$$\nu_\mu + n \to \mu^- + p$$

is observed experimentally, while the apparently similar reaction

$$\nu_\mu + n \to e^- + p$$

which violates both L_e and L_μ conservation, is not.

Finally, conservation laws explain the stability of the electron and the neutrinos. The electron is stable because electric charge is conserved in all interactions and the electron is the lightest charged particle. Hence decays to lighter particles which satisfy all other conservation laws, like $e^- \to \nu_e + \gamma$, are necessarily forbidden by electric charge conservation. In the same way, lepton number conservation implies that the lightest particles with non-zero values of the three lepton numbers – the three neutrinos – are stable, whether they have zero masses or not.

3.3 Neutrinos

The existence of the *electron neutrino* ν_e was first postulated by Pauli in 1930. He did this in order to understand the observed β-decays

$$(Z,N) \to (Z+1, N-1) + e^- + \bar{\nu}_e$$

and

$$(Z,N) \to (Z-1, N+1) + e^+ + \nu_e$$

The neutrinos and antineutrinos emitted in these decays are not observed experimentally, but are inferred from energy and angular momentum conservation. In the case of energy, if the

antineutrino were not present in the first of the reactions, the energy E_e of the emitted electron would be equal to the difference in rest energies of the two nuclei

$$E_e = \Delta M c^2 = \left[M(Z,N) - M(Z+1, N-1)\right]c^2$$

where for simplicity we have neglected the extremely small kinetic energy of the recoiling nucleus $(Z+1, N-1)$. However, if the antineutrino is present, the electron energy would not be unique, but would lie in the range

$$m_e c^2 \le E_e \le (\Delta M - m_{\bar{\nu}_e})c^2$$

depending on how much of the kinetic energy released in the decay is carried away by the neutrino. Experimentally, the observed energies span the whole of the above range and a measurement of the energy of the electron near its maximum value of $E_e = (\Delta M - m_{\bar{\nu}_e})c^2$ determines the neutrino mass. The most accurate results come from tritium decay and are compatible with zero mass neutrinos. When experimental errors are taken into account, the experimentally allowed range is

$$0 \le m_{\bar{\nu}_e} < 3eV/c^2 \approx 6\text{x}10^{-6} m_e$$

We will discuss this experiment in more detail later in Section 7, after we have considered the theory of β-decay.

The masses of both ν_μ and ν_τ can similarly be directly inferred from the e^- and μ^- energy spectra in the leptonic decays of muons and tauons using energy conservation. The results from these and other decays show that the neutrino masses are very small compared with the masses of the associated charged leptons; and again they are consistent with zero. The present limits are given in Table 3.1.

Small neutrino masses, compatible with the above limits, can be ignored in most circumstances, and there are theoretical attractions in assuming neutrino masses are precisely zero, as is done in the standard model. However, we will show in the following section that there is now strong evidence for physical phenomena that could not occur if the neutrinos had exactly zero mass. The consequences of neutrinos having small masses have therefore to be taken seriously.

Neutrinos can only be detected with extreme difficulty. For example, electron neutrinos and antineutrinos can in principle be detected by observing the *inverse β–decay* processes

$$\nu_e + n \to e^- + p$$

and

$$\bar{\nu}_e + p \to e^+ + n$$

However, because neutrinos only interact via the weak interaction, the probability for these and other processes to occur is extremely small. In particular, the neutrinos and antineutrinos emitted in β-decays, with energies of order 1MeV, have mean free paths in matter of order 10^6 km. Nevertheless, if the neutrino flux is intense enough and the detector is large enough, the reactions can be observed. In particular, uranium fission fragments are neutron rich, and decay by electron emission to give an antineutrino flux which can be of order $10^{17}\,\mathrm{m^{-2}s^{-1}}$ or more in the vicinity of a nuclear reactor. These antineutrinos will occasionally interact with protons in a large detector, enabling examples of the inverse β–decay reaction to be observed. Electron neutrinos were first detected in this way in a classic experiment by Reines and Cowan in 1959, and their interactions have been studied in considerably more detail since.

The mu neutrino ν_μ has been detected using the reaction $\nu_\mu + n \to \mu^- + p$ and other reactions. In this case, well-defined, high energy mu neutrino beams can be created in the laboratory by exploiting the decay properties of particles called *pions*, which we have mentioned briefly in Sec 1 and which we will meet in more detail presently. The probability of neutrinos interacting with matter increases rapidly with energy, and for large detectors, neutrino events initiated by such beams are so copious that they have become an indispensable tool in studying both the fundamental properties of weak interactions and the internal structure of the proton. Finally, in 2000, a few examples of tau neutrinos were reported, so that almost 70 years after Pauli first suggested the existence of a neutrino, all three types have been directly detected.

3.4 Neutrino mixing and oscillations

Neutrinos are assumed to have zero mass in the standard model, although the model can be extended to accommodate small, non-zero masses. However, we have seen that from the β-decay of tritium there is evidence for a non-zero mass. A phenomenon that can occur if neutrinos have non-zero masses is *neutrino mixing*. This arises if we assume that the observed neutrino states ν_e, ν_μ and ν_τ, which take part in weak interactions, i.e. the states which couple to electrons, muons and tauons, respectively, are not eigenstates of mass, but instead are linear combinations of three other states ν_1, ν_2 and ν_3 that do have definite masses m_1, m_2 and m_3. For simplicity we will consider the case of mixing between just two states:

$$\nu_e = \nu_1 \cos\alpha + \nu_2 \sin\alpha$$

and

$$\nu_\mu = -\nu_1 \sin\alpha + \nu_2 \cos\alpha$$

Here α is a *mixing angle* that must be determined from experiment. If $\alpha \neq 0$ then some interesting predictions follow.

Measurement of the mixing angle may be done in principle by studying the phenomenon of *neutrino oscillation*. When, for example, an electron neutrino is produced with momentum p at time $t = 0$, the ν_1 and ν_2 components will have slightly different energies E_1 and E_2 due to their slightly different masses. In quantum mechanics, their associated waves will therefore have slightly different frequencies, giving rise to phenomena somewhat akin to the "beats" heard when two sound waves of slightly different frequency are superimposed. As a result of these, one finds that the original beam of electron neutrinos develops a muon neutrino component whose intensity oscillates as it travels through space, while the intensity of the neutrino electron beam itself is correspondingly reduced.

This effect follows from simple quantum mechanics. To illustrate this we will consider an electron neutrino produced with momentum $\mathbf{p}$ at time $t = 0$. The initial state is therefore

$$\psi(\nu_e, \mathbf{p}) = \psi(\nu_1, \mathbf{p})\cos\alpha + \psi(\nu_2, \mathbf{p})\sin\alpha$$

but after a time t this will become

$$a_1(t)\psi(\nu_1, \mathbf{p})\cos\alpha + a_2(t)\psi(\nu_2, \mathbf{p})\sin\alpha$$

where

$$a_i(t) = e^{-iE_i t/\hbar} \qquad (i = 1,2)$$

are the usual oscillating energy factors associated with any quantum mechanical stationary state. For $t \neq 0$, this linear combination does not correspond to a pure electron neutrino state, but can be written as a linear combination

$$A(t)\psi(\nu_e, \mathbf{p}) + B(t)\psi(\nu_\mu, \mathbf{p})$$

where the muon neutrino state is

$$\psi(\nu_\mu, \mathbf{p}) = -\psi(\nu_1, \mathbf{p})\sin\alpha + \psi(\nu_2, \mathbf{p})\cos\alpha$$

The functions A(t) and B(t) are found by substituting the inverses of the expressions for $\psi(\nu_e, \mathbf{p})$ and $\psi(\nu_\mu, \mathbf{p})$ and are

$$A(t) = a_1(t)\cos^2\alpha + a_2(t)\sin^2\alpha$$

and

$$B(t) = \sin\alpha\cos\alpha\left[a_2(t) - a_1(t)\right]$$

The probability of finding a muon neutrino is then

$$P(\nu_e \to \nu_\mu) = \left|B(t)^2\right| = \sin^2(2\alpha)\sin^2\left[(E_2 - E_1)t/2\hbar\right]$$

and thus oscillates with time, while the probability of finding an electron neutrino is reduced by a corresponding oscillating factor. Similar effects are predicted if instead we start from muon neutrinos. In both cases the oscillations vanish if the mixing angle is zero, or if the neutrinos have equal masses, and hence equal energies, as can be seen explicitly from the equations. In particular, such oscillations are not possible if the neutrinos both have zero masses.

Attempts to detect neutrino oscillations rest on the fact that electron neutrinos can produce electrons via reactions like

$$\nu_e + n \to e^- + p$$

but cannot produce muons or tauons, whereas muon neutrinos can produce muons, via reactions like

$$\nu_\mu + n \to \mu^- + p$$

but not electrons or tauons. [If lepton numbers are not absolutely conserved these reactions would still be expected to be dominant.] In addition, the time t is determined by the distance of the neutrino detector from the source of the neutrinos, since their momenta are always much greater than their possible masses, and they travel at approximately the speed of light. Hence, for example, if we start with a beam of muon neutrinos the yield of electrons and/or muons observed in a detector should vary with its distance from the source of the neutrinos, if appreciable oscillations occur. In practice, oscillations at the few percent level are very difficult to detect for experimental reasons that we will not discuss here.

In 1998 clear evidence for the existence of neutrino oscillations was obtained from observations on *atmospheric neutrinos* by the giant *Super Kamiokande* detector in Japan. When cosmic ray protons collide with atoms in the upper atmosphere, they create many pions, which in turn create neutrinos mainly by the decay sequences

$$\pi^- \to \mu^- + \bar{\nu}_\mu \,, \qquad \pi^+ \to \mu^+ + \nu_\mu$$

and

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \,, \qquad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

From this, one would naively expect to see two muon neutrinos for every electron neutrino detected. However, the ratio was observed to be about 1.3 to 1 on average, suggesting that the muon neutrinos produced might be oscillating into other species. Clear confirmation for this was found by exploiting the fact that the detector measured the direction of the detected neutrinos to study the azimuthal dependence of the effect. In particular, one can compare the measured flux from neutrinos produced in the atmosphere directly above the detector, which have a short flight path before detection, with those incident from directly below, which have traveled a long way through the earth before detection, and so have had plenty of time to oscillate (perhaps several cycles). Experimentally, it was found that the yield of electron neutrinos from above and below were the same within errors and consistent with expectation for no oscillations. However, while the yield of muon neutrinos from above accorded with the expectation for no significant oscillations, the flux of muon neutrinos from below was a factor of about two lower. This is rather clear evidence for muon neutrino oscillations, presumably into tauon neutrinos which, for the neutrino energies concerned, cannot be detected by Super Kamiokande.

The existence of neutrino oscillations, and by implication non-zero neutrino masses, is now generally accepted on the basis of the above and other evidence. However the details, including the values of the neutrino mass differences and the various mixing angles involved, remain to be resolved and this will be done in a number of experiments that will detect oscillations directly using prepared neutrino beam and making measurements at great distances from their origin. One such experiment is called MINOS and members of our own particle physics group are involved in this, which aims to measure these parameters in the next few years.

What are the consequences of these results for the standard model? The observation of oscillations does not lead to a measurement of the neutrino masses, only (squared) mass differences, but combined with the tritium experiment, it would be natural to assume that neutrinos all had very small masses, with the mass differences being of the same order-of-magnitude as the masses themselves. The standard model can be modified to accommodate such small masses, although methods for doing this is are not without their own problems. I will mention one way at the end of Sec 6 when I briefly discuss the general problem of how masses arise in the standard model. There are also consequences for lepton number conservation. In the simple mixing model above, total lepton number could still be conserved, but individual lepton numbers would not. However, there are other theoretical descriptions of neutrino oscillations and this is an open question. A definitive answer would be to detect *neutrinoless double β-decay*, such as

$$^{76}Ge \to {}^{76}Se + 2e^-$$

as the final state contains two electrons, but no antineutrinos. A very recent experiment claims to have detected this decay, but the result is not universally accepted and at present ‘the jury is still out’. Experiments planned for the next few years should settle this very important question.

3.5 Universal lepton interactions; numbers of neutrinos

The three neutrinos have similar properties, but the three charged leptons are strikingly different. For example: the magnetic moment of the electron is roughly 200 times greater than that of the muon; high energy electrons are mostly stopped by 1 cm of lead, while muons are the most penetrating form of radiation known, apart from neutrinos; and the tauon lifetime is many orders of magnitude smaller than the muon lifetime, while the electron is stable. It is therefore a remarkable fact that all experimental data are consistent with the assumption that the interactions of the electron and its associated neutrino are identical with those of the muon and its associated neutrino and of the tauon and its neutrino, *provided the mass differences are taken into account.* This property, called *universality*, can be verified with great precision, because we have a precise theory of electromagnetic and weak interactions, which enables us to predict the mass dependence of all observables.

For example, when we discuss experimental methods in Section 4, we will show that the *radiation length*, which is a measure of how far a charged particle travels through matter before losing a certain fraction of its energy by radiation, is proportional to the squared mass of the radiating particle. Hence it is about $4\text{x}10^4$ times greater for muons than for electrons, explaining their much greater penetrating power in matter. As another example, we have seen that the rates for weak β-decays are extremely sensitive to the kinetic energy released in the decay (recall the enormous variation in the lifetimes of nuclei decaying via β-decay). The ratio of the decay rates Γ for muon and tauon leptonic decays is predicted, from universality and taking account of the different energy releases, to be

$$\frac{\Gamma(\tau^- \to e^- + \bar{\nu}_e + \nu_\tau)}{\Gamma(\mu^- \to e^- + \bar{\nu}_e + \nu_\mu)} = 1.34\text{x}10^6$$

This is excellent agreement with experiment and accounts very well for the huge difference between the tauon and muon lifetimes when the other decay modes of the tauon are also taken into account.

The above are just some of the most striking manifestations of the universality of lepton interactions. More generally, the three generations of leptons tell not three stories, but in all essential points, one story three times.

A question that arises naturally is whether there are more generations of leptons, with identical interactions, waiting to be discovered. This question has been answered, under reasonable assumptions, by an experimental study of the decays of the Z^0 boson. This particle, which has a mass of $91\text{GeV}/c^2$, is one of the two gauge bosons associated with the weak interaction. It decays, among other final states, to neutrino pairs

$$Z^0 \to \nu_\ell + \bar{\nu}_\ell \qquad (\ell = e, \mu, \tau)$$

If we assume universal lepton interactions and neutrino masses which are small compared to the mass of the Z^0, then the decay rates to a given neutrino pair will all be equal and thus

$$\Gamma_{neutrinos} = \Gamma_{\nu_e} + \Gamma_{\nu_\mu} + \Gamma_{\nu_\tau} + ... = N_\nu \Gamma_\nu$$

where N_ν is the number of neutrino species and Γ_ν is the decay rate to any given pair of neutrinos. The measured total decay rate may then be written

$$\Gamma_{total} = \Gamma_{hadrons} + \Gamma_{leptons} + \Gamma_{neutrinos}$$

where the first two terms on the right are the measured decay rates to hadrons and charged leptons, respectively. Although the rate to neutrinos Γ_ν is not directly measured, it can be calculated in the standard model and so using data the value of N_ν may be found. The result is consistent with the expectation for three, but not four, neutrino species. Only three generations of leptons can exist, if we assume universal lepton interactions and exclude very large neutrino masses.

Why there are just three generations of leptons, and not less or more, remains a mystery.

3.6 Evidence for quarks

We turn now to the strongly interacting particles – the quarks and their bound states, the hadrons. These also interact by the weak and electromagnetic interactions, although such effects can often be neglected compared to the strong interactions. To this extent we are entering the realm of "strong interaction physics".

Several hundred hadrons (not including nuclei) have now been observed, all with zero or integer electric charges: $0, \pm 1$, or ± 2 in units of e. They are all bound states of the fundamental spin-1/2 quarks, whose electric charge is either +2/3 or –1/3, and/or antiquarks, with charges –2/3 or +1/3. The quarks themselves have never been directly observed as single, free particles, but there is compelling evidence for their existence. The evidence comes from three main areas: *hadron spectroscopy, lepton scattering* and *jets.*

Hadron spectroscopy. This is the study of the static properties of hadrons: their masses, lifetimes and decay modes, and especially the values of their quantum numbers, including their spins, electric charges and many more. The existence and properties of quarks were first inferred from hadron spectroscopy (by Gell-Mann and Zweig in 1964) and the close correspondence between the experimentally observed hadrons and those predicted by the quark model, which we will examine in more detail later, remains one of the strongest reasons for our belief in the existence of quarks.

Lepton scattering. It was mentioned in the first lecture of this course that in the late 1960s experiments were performed where high-energy leptons (electrons, muons and neutrinos) were scattered from protons and neutrons. In much the same way as Rutherford deduced the existence of the nucleus in atoms, the large-angle scattering revealed the existence of point-like entities within the nucleons, which we now identify as quarks. These experiments will be discussed in Section 5.7.

Jets. High-energy collisions can cause the quarks within hadrons, or newly created quark-antiquark pairs, to fly apart from each other with very high energies. Before they can be observed, these quarks are converted by relatively gentle interactions (a process referred to as *fragmentation*) into jets of hadrons, whose production rates and angular distributions reflect those of the quarks from which they originated. They were first clearly identified in electron-positron collisions at the DESY laboratory in Hamburg in 1979, and an example of a "two-jet" event observed is shown in Fig.3.3.

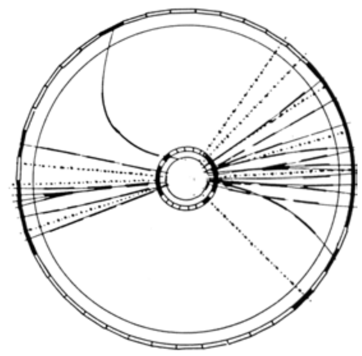

Fig.3.3 Two-jet event in e^+e^- collisions

This picture is a computer reconstruction of an end view along the beam direction; the solid lines indicate the reconstructed charged particle trajectories taking into account the known magnetic field, which is also parallel to the beam direction; the dotted lines indicate the reconstructed trajectories of neutral particles, which were detected outside the chamber by other means.

The production rate and angular distribution of the observed jets closely matches that of quarks produced in the reaction

$$e^+ + e^- \to q + \bar{q}$$

by the mechanism of Fig.3.4. Such jets have now been observed in many reactions, and are the closest thing to a quark "track" we are ever likely to see

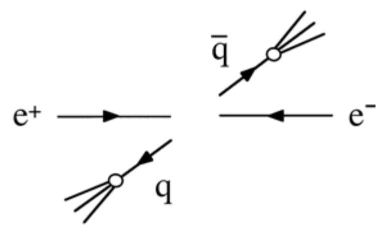

Fig.3.4 Mechanism for two-jet production in e^+e^- annihilation reaction

The failure to detect free quarks is not an experimental problem. Firstly, free quarks would be easily distinguished from other particles by their fractional charges and their resulting ionization properties. (We will see in Sec 4 that ionization energy losses in matter are proportional to the square of the charge.) Secondly, electric charge conservation implies that a fractionally charged particle cannot decay to a final state composed entirely of particles with integer electric charges. Hence the lightest fractionally charged particle, i.e. the lightest free quark, would be stable and so presumably easy to observe. Finally, some of the quarks are not very massive (see below) and because they interact by the strong interaction, one would expect free quarks to be copiously produced in, for example, high-energy proton-proton collisions. However, despite careful and exhaustive searches in ordinary matter, in cosmic rays and in high-energy collision products, free quarks have never been observed. The conclusion – that quarks exist solely within hadrons and not as isolated free particles – is called *confinement.*

The modern theory of strong interactions, called *quantum chromodynamics*, (which we will discuss in Section 5) offers at least a qualitative account of confinement, although the details elude us due to the extreme difficulty of performing accurate calculations. In what follows, we shall assume confinement and use the properties of quarks to interpret the properties of hadrons. We start with the basic properties of quarks.

3.7 Properties of quarks

Six distinct types, or *flavours*, of spin-1/2 quarks are now known to exist. Like the leptons, they occur in pairs, or *generations*, denoted

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Each generation consists of a quark with charge +2/3 (u, c, or t) together with a quark of charge –1/3 (d, s, or b), in units of e. They are called the *down* (d), *up* (u), *strange* (s), *charmed* (c), *bottom* (b) and *top* (t) quarks. The quantum numbers associated with the *s, c, b* and *t* quarks are called *strangeness, charm, beauty* and *truth*, respectively. The antiquarks are denoted

$$\begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}, \quad \begin{pmatrix} \bar{s} \\ \bar{c} \end{pmatrix}, \quad \begin{pmatrix} \bar{b} \\ \bar{t} \end{pmatrix}$$

with charges +1/3 ($\bar{d}$, $\bar{s}$, or $\bar{b}$) and –2/3 ($\bar{u}$, $\bar{c}$, or $\bar{t}$).

Approximate quark masses are given in Table 3.2. Except for the top quark, these masses are inferred indirectly from the observed masses of their hadron bound states, together with models of quark binding. An analogy would be to deduce the mass of nucleons from the masses of nuclei via a model such as the liquid drop model.

Table 3.2 Properties of quarks. All have spin-1/2. Masses are given units of GeV/c^2. The antiparticles (not shown) have the same masses as their associated particles, but the electric charges (Q) are reversed in sign. In the major decay modes X denotes other particles.

Name	Symbol	Mass	Q	Lifetime (s)	Major decays
down	d	$m_d \approx 0.3$	–1/3		
up	u	$m_u \approx m_d$	2/3		
strange	s	$m_s \approx 0.5$	–1/3	$10^{-8} - 10^{-10}$	$s \to u + X$
charmed	c	$m_c \approx 1.5$	2/3	$10^{-12} - 10^{-13}$	$c \to s + X$ $c \to d + X$
bottom	b	$m_b \approx 4.5$	–1/3	$10^{-12} - 10^{-13}$	$b \to c + X$
top	t	$m_t = 180 \pm 12$	2/3	$\sim 10^{-25}$	$t \to b + X$

The stability of quarks in hadrons – like the stability of protons and neutrons in atomic nuclei – is influenced by their interaction energies. However, for the *s, c* and *b* quarks these effects are small enough for them to be assigned approximate lifetimes of $10^{-8} - 10^{-10}$ s for the *s*-quark and $10^{-12} - 10^{-13}$ s for both the *c*- and *b*-quarks. The top quark is much heavier than the other quarks and its lifetime is of order 10^{-25} s. This lifetime is so short that, when top quarks are created, they decay too quickly to form observable hadrons. In contrast to the other quarks, our knowledge of the top quark is based entirely on observations of its decay products. When we talk about "the decay of quarks" we always mean that the decay takes place within a hadron, with the other bound quarks acting as "spectators". Thus, for example, neutron decay in this picture is given by the Feynman-like *quark diagram* of Fig.3.5 and no free quarks are observed.

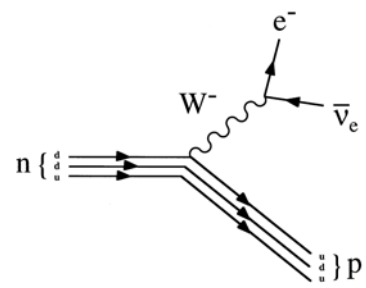

Fig.3.5 Quark diagram for the decay $n \to p e^- \bar{\nu}_e$

3.8 Quark numbers

In strong and electromagnetic interactions, quarks can only be created or destroyed as particle-antiparticle pairs. This implies, for example, that in electromagnetic processes corresponding to the Feynman diagram of Fig.3.6, the reaction $e^+ + e^- \to c + \bar{c}$, which creates a $c\bar{c}$ pair, is allowed, but the reaction $e^+ + e^- \to c + \bar{u}$ producing a $c\bar{u}$ pair, is forbidden. More generally, it implies conservation of each of the six *quark numbers*

$$N_f \equiv N(f) - N(\bar{f}) \qquad (f = u, d, s, c, b, t)$$

where $N(f)$ is the number of quarks of flavour f present and $N(\bar{f})$ is the number of $\bar{f}$-antiquarks present.

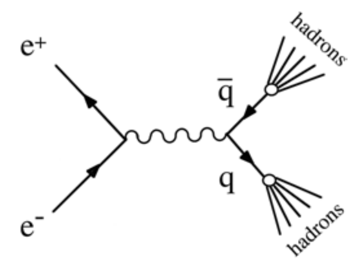

Fig.3.6 Production mechanism for the reaction $e^+e^- \to q\bar{q}$

For example, for single-particle states; $N_c = 1$ for the *c*-quark; $N_c = -1$ for the $\bar{c}$ antiquark; and $N_c = 0$ for all other particles. Similar results apply for the other quark numbers N_f, and for multi-particle states the quark numbers of the individual particles are simply added. Thus a state containing the particles u, u, d, has $N_u = 2$, $N_d = 1$ and $N_f = 0$ for the other quark numbers with $f = s, c, b, t$.

In weak interactions, more general possibilities are allowed, and only the total quark number

$$N_q \equiv N(q) - N(\bar{q})$$

is conserved, where $N(q)$ and $N(\bar{q})$ are the total number of quarks and antiquarks present, irrespective of their flavour. This is illustrated by the decay modes of the quarks themselves, some of which are listed in Table 3.2, which are all weak interaction processes, and we have seen it also in the decay of the neutron in Fig.3.5. Other example is the *main* decay mode of the charmed quark, which is

$$c \to s + u + \bar{d}$$

in which a *c*-quark is replaced by an *s*-quark and a *u*-quark is created together with a $\bar{d}$ antiquark. This clearly violates conservation of the quark numbers N_c, N_s, N_u and N_d, but the total quark number N_q is conserved.

In practice, it is convenient to replace the total quark number N_q in discussions by the *baryon number*, defined by

$$B \equiv N_q/3 = [N(q) - N(\bar{q})]/3 .$$

Like the electric charge and the lepton numbers introduced in the last section, the baryon number is conserved in *all known interactions*.

3.9 Hadrons

In principle, the properties of atoms and nuclei can be explained in terms of their proton, neutron and electron constituents, although in practice many details are too complicated to be accurately calculated. However the properties of these constituents can be determined without reference to atoms and nuclei, by studying them directly as free particles in the laboratory. In this sense atomic and nuclear physics are no longer fundamental, although they are still very interesting and important if we want to understand the world we live in.

In the case of hadrons the situation is more complicated. Their properties are explained in terms of a few fundamental quark constituents; but the properties of the quarks themselves can only be studied experimentally by appropriate measurements on hadrons. Whether desirable or not, studying quarks without hadrons is not an option.

The observed hadrons are of three types. These are the *baryons*, which have half-integer spin and are assumed to be bound states of three quarks ($3q$); the *antibaryons*, which are their antiparticles and are assumed to be bound states of three antiquarks ($3\bar{q}$); and the *mesons* which have integer spin and are assumed to be bound states of a quark and an antiquark ($q\bar{q}$). These assumptions constitute the so-called *quark model of hadrons*. The baryons and antibaryons have baryon numbers 1 and –1 respectively, while the mesons have baryon number 0. Hence the baryons and antibaryons can annihilate each other in reactions which

conserve baryon number to give mesons or, more rarely, photons or lepton-antilepton pairs, in the final state. Some examples of baryons and mesons, together with their quark compositions, are shown in Table 3.3.

Table 3.3 Some examples of baryons and mesons, with their quark compositions and major decay modes. Masses are in MeV/c^2.

Particle	Mass	Lifetime (s)	Major decays
$\pi^+(u\bar{d})$	140	$2.6\text{x}10^{-8}$	$\mu^+\nu_\mu$ (~100%)
$\pi^0(u\bar{u}, d\bar{d})$	135	$8.4\text{x}10^{-17}$	$\gamma\gamma$ (~100%)
$K^+(u\bar{s})$	494	$1.2\text{x}10^{-8}$	$\mu^+\nu_\mu$ (64%) $\pi^+\pi^0$ (21%)
$K^{*+}(u\bar{s})$	892	$\sim 1.3\text{x}10^{-23}$	$K^+\pi^0$, $K^0\pi^+$ (~100%)
$D^-(d\bar{c})$	1869	$1.1\text{x}10^{-12}$	Several seen
$B^-(b\bar{u})$	5278	$1.6\text{x}10^{-12}$	Several seen
$p(uud)$	938	Stable	None
$n(udd)$	940	887	$pe^-\bar{\nu}_e$ (100%)
$\Lambda(uds)$	1116	$2.6\text{x}10^{-10}$	$p\pi^-$ (64%) $n\pi^0$ (36%)
$\Delta^{++}(uuu)$	1232	$\sim 0.6\text{x}10^{-23}$	$p\pi^+$ (100%)
$\Omega^-(sss)$	1672	$0.8\text{x}10^{-10}$	ΛK^- (68%) $\Xi^0\pi^-$ (24%)
$\Lambda_c^+(udc)$	2285	$2.1\text{x}10^{-13}$	Several seen

The lightest known baryons are the proton and neutron with the quark compositions

$$p = uud , \qquad n = udd$$

These particles have been familiar as constituents of atomic nuclei since the 1930s. The birth of particle physics as a new subject, distinct from atomic and nuclear physics, dates from 1947, when hadrons other than the neutron and proton were first detected. These were the *pions* and the *kaons*, discovered in cosmic rays by groups in Bristol and Manchester respectively.

Pions have been briefly mentioned in earlier lectures. Their discovery was not totally unexpected, since Yukawa had famously predicted their existence and their approximate masses in 1935, in order to explain the observed range of nuclear forces. Briefly, this consisted of finding what mass particle was needed in the Yukawa potential to give the observed range of the nuclear force. It turned out to be about 120 MeV/c^2, a little less than the observed mass of the pion, and after some false signals a particle with this mass and the right properties was discovered. There are three types of pion, denoted $\pi^\pm(140)$, $\pi^0(135)$, where here and in what follows we give the hadron masses in brackets in units of MeV/c^2 and use a superscript to indicate the electric charge in units of *e*. They are the lightest known mesons and have the quark compositions

$$\pi^+ = u\bar{d} , \quad \pi^0 = u\bar{u}, d\bar{d} , \quad \pi^- = d\bar{u}$$

While the charged pions have a unique composition, the neutral pion is composed of both $u\bar{u}$ and $d\bar{d}$ pairs in equal amounts. Nowadays, all the pions can be copiously produced in high-energy collisions at accelerators by strong interaction processes such as

$$p + p \to p + n + \pi^+$$

In contrast to the discovery of the pions, the discovery of the kaons was totally unexpected, and they were almost immediately recognized as a completely new form of matter, because they had supposedly "strange" properties. Eventually (1954) it was realized that these properties were precisely what would be expected if kaons had non-zero values of a hitherto unknown quantum number, called *strangeness,* which was conserved in strong and electromagnetic interactions, but not necessarily conserved in weak interactions. Particles with non-zero strangeness were christened *strange particles,* and with the advent of the quark model in 1964, it was realized that strangeness S was, apart from a sign, the strange quark number introduced in Section 3.8, i.e.

$$S = -N_s$$

Kaons are the lightest strange mesons, with the quark compositions:

$$K^+(494) = u\bar{s} \;, \qquad K^0(498) = d\bar{s}$$

where K^+ and K^0 have S = +1 and K^- and $\bar{K}^0$ have S = –1, while the lightest strange baryon is the *lambda,* with the quark composition

$$\Lambda = uds$$

Subsequently, hadrons containing c and b quarks have also been discovered, with non-zero values of the *charm* and *beauty* quantum numbers defined by

$$C \equiv N_c \equiv N(c) - N(\bar{c}) \;\text{ and }\; \tilde{B} \equiv -N_b \equiv -N(b) - N(\bar{b})$$

The above examples illustrate just some of the different combinations of quarks that form baryons or mesons. To proceed more systematically one could, for example, construct all the mesons states of the form $q\bar{q}$ where q can be any of the six quark flavours. The simplest such states would have the spins of the two quarks antiparallel with no orbital angular momentum between them, and so have spin-0. If, for simplicity, we such consider those states composed of u, d and s quarks, you can easily find that the nine bosons have quantum numbers which may be identified with the observed mesons (K^0, K^+), $(\bar{K}^0, K^-)$, $(\pi^\pm, \pi^0)$ and two neutral particles, which are called η and η'. This can be extended to the lowest lying baryon states qqq and also to all six quark flavours. It is a remarkable fact that the states observed experimentally agree with those predicted by the simple combinations qqq, $\bar{q}\bar{q}\bar{q}$ and $q\bar{q}$, and there is no convincing evidence for states corresponding to other combinations. This was one of the original pieces of evidence for the existence of quarks and remains one of the strongest pieces of evidence in favour of the quark model.

For many of these quark combinations there exist not one, but many states. This is illustrated in Fig.3.7 which shows all the known $u\bar{d}$ states with masses below $1.5\,\mathrm{GeV}/c^2$.

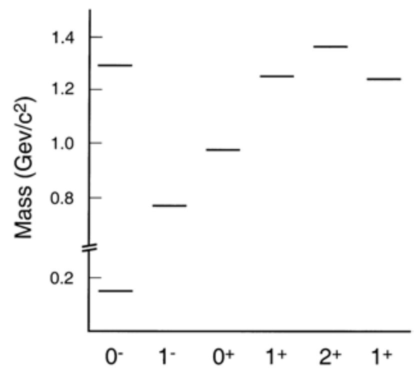

Fig.3.7 Spectrum of $u\bar{d}$ quark states below $1.5\,\mathrm{GeV}/c^2$

Each of these states is labeled by its spin and by its *parity P*, which is a quantum mechanical observable related to the behaviour of the state under a mirror reflection, i.e $\mathbf{r} \to -\mathbf{r}$ in the wavefunction. If the wavefunction remains unaltered, then the parity is +1; if it changes sign then the parity is –1. (We will discuss this in more detail when we consider weak interactions later in Section 6.) The notation $1^\pm$ is used to indicate a particle of spin-1 with negative parity, and so on. The lowest lying state shown in Fig.3.7 has spin-parity $0^\pm$ and is the π^+ meson discussed above. It can be regarded as the "ground state" of the $u\bar{d}$ system. Here the spins of the quark constituents are anti-aligned to give a total spin $\mathbf{S} = \mathbf{0}$ and there is no orbital angular momentum $\mathbf{L}$ between the two quarks, so that the total angular momentum, which we identify as the spin of the hadron, is $\mathbf{J} = \mathbf{L} + \mathbf{S} = \mathbf{0}$. The other "excited" states can have different spin-parities depending on the different states of motion of the quarks within the hadron. These are *resonances* and they usually decay by the strong interaction, with very short lifetimes, of order $10^{-23}\,\mathrm{s}$. The mass distribution of their decay products is described by the Breit-Wigner formula we met in an earlier lecture. It is part of the triumph of the quark model that it successfully accounts for the excited states of the various quark systems, as well as their ground states, when the internal motion of the quarks is properly taken into account.

Hadrons have typical radii r of order 1 fm, with an associated time scale r/c of order $10^{-23}\,\mathrm{s}$. The vast majority are highly unstable resonances, corresponding to excited states of the various quark systems, and decay to lighter hadrons by the strong interaction with lifetimes of this order. A typical example is the $K^{*+}(890) = u\bar{s}$ resonance, which decays to $K^+\pi^0$ and $K^0\pi^+$ final states with a lifetime of $1.3\mathrm{x}10^{-23}\,\mathrm{s}$. The quark description of the process $K^{*+} \to K^0 + \pi^+$, for example, is

$$u\bar{s} \to d\bar{s} + u\bar{d} \;.$$

From this we see that the final state contains the same quarks as the initial state, plus an additional $d\bar{d}$ pair, so that the quark numbers N_u and N_d are separately conserved. This is characteristic of strong and electromagnetic processes, which are only allowed if all the quark numbers N_u, N_d, N_s, N_c, and N_b are separately conserved.

Since leptons and photons do not have strong interactions, hadrons can only decay by the strong interaction if lighter states composed solely of other hadrons exist with the same quantum numbers. While this is possible for the majority of hadrons, it is not in general possible for the lightest state corresponding to any given quark combination. These hadrons, which cannot decay by strong interactions, are long-lived on a timescale of order 10^{-23} s and are often called *stable particles*. It is more accurate to call them *long-lived particles*, because except for the proton they are not absolutely stable, but decay by either the electromagnetic or weak interaction.

The proton is stable because it is the lightest particle with non-zero baryon number and baryon number is conserved in all known interactions. A few of the other long-lived hadrons decay by electromagnetic interactions to final states which include photons. These decays, like the strong interaction, conserve all the individual quark numbers. An example of this is the neutral pion, which has $N_u = N_d = N_s = N_c = N_b = 0$ and decays by the reaction

$$\pi^0(u\bar{u},\, d\bar{d}) \to \gamma + \gamma$$

with a lifetime of $0.8\text{x}10^{-16}$ s. However, most of the long-lived hadrons have non-zero values for at least one of the quark numbers, and can only decay by the weak interaction, which can violate quark number conservation. For example, the positive pion decays with a lifetime of $2.6\text{x}10^{-8}$ s by the reaction

$$\pi^+ \to \mu^+ + \nu_\mu$$

while the $\Lambda(1116) = uds$ baryon decays mainly by the reaction

$$\Lambda \to p + \pi^-$$

with a lifetime of $2.6\text{x}10^{-10}$ s. The quark interpretations of these reactions are

$$(u\bar{d}) \to \mu^+ + \nu_\mu$$

in which a *u*-quark annihilates with a $\bar{d}$-antiquark, violating both N_u and N_d conservation; and for lambda decay

$$sud \to uud + d\bar{u}$$

in which an s quark turns into a u quark and a $u\bar{d}$ pair is created, violating N_d and N_s conservation.

We see from the above that the strong, electromagnetic or weak nature of a given hadron decay can be determined by inspecting quark numbers. The resulting lifetimes can then be summarized as follows. Strong decays lead to lifetimes that are typically of order 10^{-23} s. Electromagnetic decay rates are suppressed by powers of the fine structure constant α relative to strong decays, leading to observed lifetimes in the range $10^{-16} - 10^{-21}$ s. Finally, weak decays give longer lifetimes, which depend sensitively on the characteristic energy of the decay. A useful measure of this characteristic energy is the Q-*value*, which is the kinetic

energy released in the decay of the particle at rest. For neutron decay, $n \to p + e^- + \bar{\nu}_e$, the Q-value

$$Q = m_n - m_p - m_e - m_{\bar{\nu}_e} = 0.79\,\text{MeV}$$

is very small, leading to a lifetime of about $10^3\,s$. However, $Q-$values of order $10^2 - 10^3$ MeV are more typical, leading to lifetimes in the range $10^{-7} - 10^{-13}$ s. Thus hadron decay lifetimes are reasonably well understood and span some 27 orders of magnitude, from about 10^{-24}s to about 10^3s. The typical ranges corresponding to each interaction are summarised in Table 3.4 .

Table 3.4 Typical lifetimes of hadrons decaying by the three interactions.

Interaction	Lifetimes (s)
Strong	$10^{-22} - 10^{-24}$
Electromagnetic	$10^{-16} - 10^{-21}$
Weak	$10^{-7} - 10^{-13}$

3.10 Flavour independence and charge multiplets

Flavour independence is one of the most fundamental properties of the strong interaction. It is the statement that the strong force between two quarks at a fixed distance apart is independent of which quark flavours u, d, s, c, b, t are involved. Thus, for example, the strong forces between us and ds pairs are identical. The same principle applies to quark-antiquark forces, which are, however, not identical to quark-quark forces. Flavour independence does not apply to the electromagnetic interaction, since the quarks have different electric charges, but compared to the strong force between quarks, the electromagnetic force is a small correction. In addition, in applying flavour independence one must take account of the quark mass differences, which can be non-trivial. However, there are cases where these corrections are small or easily estimated, and the phenomenon of flavour independence is plain to see.

One consequence of this is the striking observation that hadrons occur in families of particles with approximately the same masses, called *charge multiplets*. Within a given family, all particles have the same spin-parity and the same strangeness, charm and beauty, but differ in their electric charges. Examples are the triplet of pions, (π^+, π^0, π^-), and the nucleon doublet (p, n). This behaviour reflects an approximate symmetry between *u* and *d* quarks. This arises because these two quarks have the same mass, apart from a small correction

$$m_d - m_u = (3 \pm 1)\,\text{MeV}/\text{c}^2$$

so that in this case, mass corrections can to a good approximation be neglected. For example, consider the case of the proton and neutron, with quark contents

$$p(938) = uud\,, \qquad n(940) = udd\,.$$

If we neglect the small mass difference between the *u* and *d* quarks and also the electromagnetic interactions, which is equivalent to setting all electric charges to zero, so that the forces acting on the *u* and *d* quarks are exactly equal, then replacing the *u* quark by a *d* quark in the proton would produce a “neutron” which is essentially identical to the proton.

Another example is the K meson doublet

$$K^{+}(494) = u\bar{s}\ , \qquad K^{0}(498) = d\bar{s}$$

where again, interchanging a u and d quark interchanges K^{+} and K^{0}. Of course the symmetry is not exact because of the small mass difference between the u and d quarks and because of the electromagnetic forces, and it is these that lead to the small differences in mass within multiplets. The symmetry between u and d quarks is called *isospin symmetry* and greatly simplifies the interpretation of hadron physics. Flavour independence of the strong forces between u and d quarks also leads directly to the *charge independence of nuclear forces*, e.g the proton-proton force is equal to the proton-neutron force provided the two particles are in the same spin state, which is approximately verified experimentally, as we mentioned when discussing the nuclear force.

A case where the mass differences between quarks are large, but relatively easily taken into account, is the comparison of the $c\bar{c}$ and $b\bar{b}$ quark systems. These are called *charmonium* and *bottomium*, respectively, by analogy with *positronium*, which is the bound state of an electron and a positron. They are important because, in this case, the quarks are so heavy that they move slowly enough within the resulting hadrons to be treated non-relativistically to a first approximation. This means that the rest energies of the bound states, and hence their masses, can be calculated from the static potential between the quarks in exactly the same way that the energy levels in the hydrogen atom (and positronium) are calculated from the Coulomb potential. In this case, however, the procedure is reversed, with the aim of determining the form of the static potential from the rather precisely measured energies of the bound states. To cut a long story short, one finds that the potentials required to describe the system are the same within the reasonably small uncertainties of the method, confirming again the flavour independence of the strong force.