2. NUCLEAR PHENOMENOLOGY
We turn now to start examining what we learn from experiments, beginning with some basic facts about nuclei. But before that we have to introduce some notation.

2.1 Notation
Nuclei are specified by:

Z – Atomic Number = the number of protons
N – Neutron Number = the number of neutrons
A – Mass Number = the number of nucleons

The charge on the nucleus is $+Ze$, where e is the absolute value of the electric charge on the electron. Why the charge on the proton should be exactly the same magnitude as that on the electron is a puzzle of very long-standing. A solution to this is suggested by some as yet unproved, but widely believed, theories of particle physics, which you may discuss in the 4th Yr course on Particle Physics.

Nuclei with combinations of these three numbers are called *nuclides* and are written ${}^{A}X$ or ${}^{A}_{Z}X$, where X is the chemical symbol for the element. Some other common nomenclature is:

Nuclides with the same mass number are called *isobars*
Nuclides with the same atomic number are called *isotopes*
Nuclides with the same neutron number are called *isotones*

Thus, for example, stable isotopes of carbon are ${}^{12}C$ and ${}^{13}C$, and the unstable isotope used in radiocarbon dating is ${}^{14}C$, all of which have $Z = 6$.

2.2 Masses and binding energies
The masses of the proton and neutron are known very accurately:

$$M_p = 938.272 \text{ MeV}/c^2 \qquad M_n = 939.566 \text{ MeV}/c^2$$

and it might be thought that the mass of a nucleus would be given by

$$M(Z,A) = Z\,M_p + N\,M_n$$

whereas actual mass measurements show that

$$M(Z,A) < Z\,M_p + N\,M_n$$

The mass deficit is

$$\Delta M(Z,A) = M(Z,A) - Z\,M_p - N\,M_n$$

and $-\Delta Mc^2$ is called the *binding energy B*. A commonly used quantity is the *binding energy per nucleon* B/A, which is the minimum energy required to remove a nucleon from the nucleus. This is shown schematically in Fig.2.1.

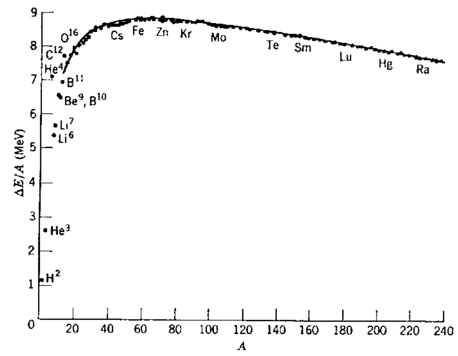

Fig.2.1 Binding energy per nucleon as a function of A for stable nuclei.

Figure 1 shows that B/A peaks at a value of 8.7 for a mass number of about 60 and thereafter falls very slowly. Over a wide range of the periodic table the binding energy per nucleon is between 7 and 9 MeV/c^2. We will discuss presently an explanation for the shape of this curve.

2.3 Nuclear forces
The basic nuclear force is that between two nucleons. The existence of stable nuclei immediately implies that overall this must be *attractive* and much stronger than the Coulomb force. However, it cannot be attractive for all separations, or otherwise nuclei would collapse in on themselves. So at very short ranges there must be a repulsive core. In lowest order the potential may be represented by a central potential, although there is also a smaller non-central part. We also know from proton-proton scattering experiments that the nucleon-nucleon force is *short-range*, of the same order as the size of the nucleus, and thus does not correspond to the exchange of gluons, as in the fundamental strong interaction. A schematic diagram of the resulting potential is shown in Fig.2.2.

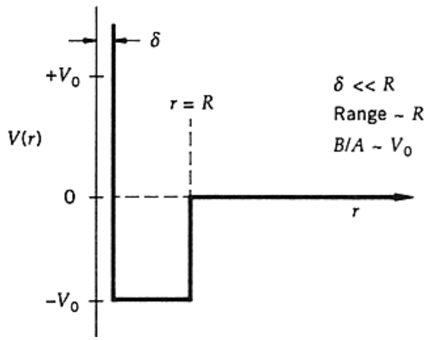

Fig.2.2 Approximate description of the nuclear potential.

A comparison of *pp* and *nn* scattering data shows that the nuclear force is *charge-symmetric* ($pp = nn$) and almost *charge-independent* ($pp = nn = pn$). Further evidence for this comes from the near equality of the energy levels of *mirror nuclei* such as ${}^{11}_{5}B$ and ${}^{11}_{6}C$, which are identical except for the substitution of one proton by one neutron. Nucleon-nucleon forces are however strongly *spin-dependent*. Thus, the force between a proton and neutron in an overall spin-1 state (i.e. with spins parallel) is strong enough to support a weakly bound state (called the *deuteron*), whereas the potential corresponding to the spin-0 state (i.e. spins antiparallel) has no bound state. Finally, nuclear forces *saturate*. This describes that fact that a nucleon in a typical nucleus experiences attractive interactions only with a limited number of the many other nucleons. The evidence for this is shown in Fig.2.1, where we see that the binding energy per nucleon is largely independent of A (except for very small values of A). If saturation did not occur, there would be $A(A-1) \approx A^2$ pairwise interaction and the binding energy per nucleon would be proportional to A, which is not observed.

Ideally one would like to be able to interpret the nuclear potential in terms of the fundamental quark-quark interactions, but nuclear physics is a long way from this yet and all one can say at present is that the attraction is due mainly to the exchange of light hadrons, particularly the pions, which we will see in Sec 3 are interpreted as quark-antiquark pairs. The repulsion seen at very short distances is largely due to the spin dependence of quark-quark forces, which depend on the total spin, just as for nucleons.

2.4 Shapes and sizes

The shape and size of a nucleus may be found from scattering experiments; i.e. a projectile is scattered from the nucleus and the angular distribution of the scattered particles examined. The interpretation is simplest in those cases where the projectile itself has no internal structure, i.e. an elementary particle, and electrons are usually used. This will tell us information about the charge distribution in the nucleus. The basic formula which describes the scattering is the *Rutherford cross-section*, which in its relativistic form may be written

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{Z^2\alpha^2(\hbar c)^2}{4E^2\sin^4(\theta/2)}$$

where E is the total energy of the initial electron and θ is the angle through which it is scattered. This formula needs to be modified in two ways before it can be used in practice. Firstly, it neglects the spins of the particles. Including the electron spin leads to the so-called *Mott cross-section*

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}\left[1-\beta^2\sin^2(\theta/2)\right]$$

where $\beta = v/c$ and v is the velocity of the initial electron. The second modification is due to the spatial extension of the nucleus, i.e. it is not an elementary point-like particle. If the spatial charge distribution within the nucleus is written $f(\mathbf{x})$ then we define the *form factor* $F(\mathbf{q}^2)$ by

$$F(\mathbf{q}^2) \equiv \frac{1}{Ze}\int e^{i\mathbf{q}.\mathbf{x}/\hbar} f(\mathbf{x})\, d^3\mathbf{x} \quad \text{with} \quad Ze = \int f(\mathbf{x})\, d^3\mathbf{x}$$

i.e. the Fourier transform of the charge distribution, where **q** is the momentum transfer for the electron – the difference between its initial and final momenta. (Strictly this formula assumes that the recoil of the target nucleus is negligible and the interaction is relatively weak so that perturbation theory may be used.) The experimental cross-section is then given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{expt} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left|F(\mathbf{q}^2)\right|^2$$

Thus, if one measure the cross-section for a fixed energy at various angles (and hence various **q**), the form factor can be extracted and in principle the charge distribution found from the inverse Fourier transform

$$f(\mathbf{x}) = \frac{Ze}{(2\pi)^3}\int F(\mathbf{q}^2)e^{-i\mathbf{q}.\mathbf{x}/\hbar}\, d^3\mathbf{q}$$

In practice the differential cross-section decreases extremely rapidly with angle, falling many orders of magnitude (see Fig.2.3) and cannot be measured over a sufficiently wide range of angles for the integral to be evaluated accurately.

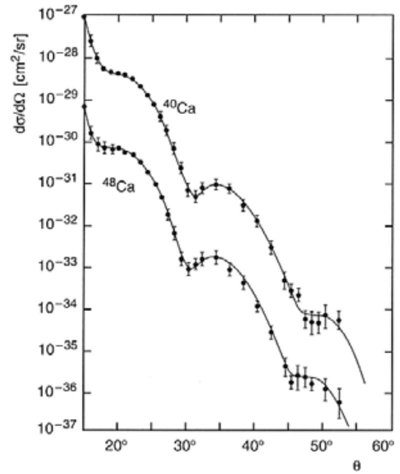

Fig.2.3 Differential cross-section as a function of angle for electrons scattered from two isotopes of calcium.

[**Aside:** The minima are due to the form factors. For example, if we assume the charge distribution is symmetric, then doing the angular part of the integration gives

$$F(\mathbf{q}^2) = \frac{4\pi\hbar}{Zeq}\int_0^\infty \rho(r)\sin\left(\frac{qr}{\hbar}\right)dr$$

If we also assume, for simplicity, that the charge distribution is a hard sphere such that

$$\begin{aligned}\rho(r) &= \text{constant}\,, \quad && r \le a \\ &= 0 && r > a\end{aligned}$$

then it is simple to carry out the integration and show that

$$F(\mathbf{q}^2) \propto \sin(b) - b\cos(b)$$

where $b \equiv qa/\hbar$. Thus there will be zeros in $F(\mathbf{q}^2)$ at solutions of $b = \tan(b)$. In practice, $\rho(r)$ is not a hard sphere and the zeros are 'softened' to dips, i.e. minima.]

Instead, a parameterised form is chosen for the charge distribution, then the form factor is calculated from the Fourier transform and a fit made to the data using the resulting expression for the differential cross-section. Some radial charge distributions for various nuclei that are obtained by this method are shown in Fig.2.4. These can be fitted by the form

$$\rho_{ch}(r) = \frac{\rho_{ch}{}^0}{1+e^{(r-c)/a}}$$

where c and a are measured to be

$$c = 1.07A^{1/3}\,\text{fm}\,; \qquad a = 0.54\,\text{fm}$$

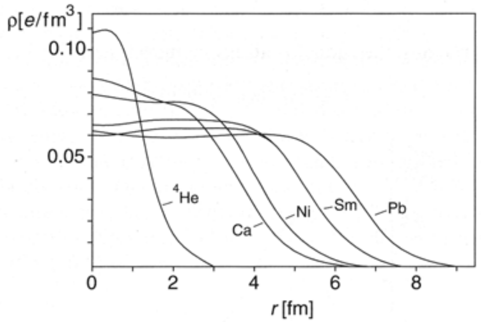

Fig.2.4 Radial charge distributions of various nuclei

From this we can deduce that the charge density is approximately constant in the nuclear interior and falls fairly rapidly to zero at the nuclear surface. The value of $\rho_{ch}{}^0$ is in the range 0.06-0.08 for medium to heavy nuclei and decreases slowly with increasing mass number.

We can also calculate the *mean square radius*,

$$\langle r^2 \rangle^{1/2} \propto \sqrt{\int r^2 \rho_{ch}(r)\,dr}$$

which for medium and heavy nuclei is approximately given by

$$\langle r^2 \rangle^{1/2} = 0.94A^{1/3}\,\text{fm}$$

The nucleus is often approximated by a homogeneous charged sphere. The radius R of this sphere is then quoted as the nuclear radius. The relation of this to the mean square radius is $R^2 = \frac{5}{3}\langle r^2 \rangle$ so that

$$R_{charge} = 1.21A^{1/3}\,\text{fm}$$

If, instead of using electrons, we use a strongly interacting particle, i.e. a hadron, as the projectile, we can probe the nuclear (i.e. matter) density of nuclei. In principle, the method follows the same steps as for the analysis of electron scattering: a form is chosen for the nuclear density, the form factor is calculated and the resulting differential cross-section compared with experiment. In practice the same mathematical form as is used as for electron scattering and is a good representation of the data. Alternatively, if one takes the presence of neutrons into account by multiplying $\rho_{ch}(r)$ by A/Z, then one finds an almost identical nuclear density in the nuclear interior for all nuclei, i.e. the decrease in $\rho_{ch}{}^0$ with increasing A is compensated by the increase in A/Z with increasing A. The interior nuclear density is given by

$$\rho_{nucl} \approx 0.17\ \text{nucleons}/\text{fm}^3$$

Likewise, the effective nuclear matter radius for medium and heavy nuclei is

$$R_{nuclear} \approx 1.2\,A^{1/3}\ fm$$

These are important results.

2.5 Liquid drop model: semi-empirical mass formula

Much of the behaviour deduced above is very similar to that of a classical liquid, where the nucleus is an incompressible liquid droplet and the nucleons play the role of individual molecules within the droplet. The analogy is not perfect of course because in discussing nucleons, quantum effects cannot be completely ignored. The liquid drop model gives rise to the *semi-empirical mass formula* which plays an important role in the discussion of nuclear stability. It is a *semi*-empirical formula because although it contains a number of constants that have to be fixed by fitting experimental data, the formula does have a theoretical basis. This consists of the two properties common to all nuclei, except those with very small A values: (1) the interior mass densities are approximately equal; and (2) their total binding energies are approximately proportional to their masses. The analogy with a classical liquid is, for drops of various sizes: (1) their interior densities are the same; and (2) their latent heats of vaporization are proportional to their masses. (The latter is the energy required to disperse the drop into its constituents and so is analogous to the binding energy.)

The semi-empirical mass formula may be written as the sum of six terms:

$$M(Z,A) = \sum_{i=0}^{5} f_i(Z,A)$$

The first of these terms is the *mass of the constituent nucleons*,

$$f_0(Z,A) = Z\,M_p + (A-Z)M_n$$

The remaining terms are various corrections, which we will write in the form a_i multiplied by a function of Z and A with $a_i > 0$.

The most important correction is the *volume* term,

$$f_1(Z,A) = -a_1 A$$

This arises from the fact the nuclear force is short range and each nucleon feels the effect of only the nucleons immediately surrounding it, independent of the size of the nucleus. This leads to the binding energy being proportional to the volume, or nuclear mass (recall the important result that the nuclear radius is proportional to $A^{1/3}$). The coefficient is negative; i.e. it increases the binding energy, as expected.

The volume term overestimates the effect of the nuclear force because nucleons at the surface are not surrounded by other nucleons. Thus the volume term and has to be corrected. This is done by the *surface* term

$$f_2(Z,A) = +a_2 A^{2/3}$$

which is proportional to the surface area (again recall that $r \propto A^{1/3}$) and decreases the binding energy. (In the classical liquid model this term would correspond to the surface tension energy.)

The *Coulomb* term accounts for the Coulomb energy of the charged nucleus, i.e. the fact that the protons repel each other. If we have a uniform charge distribution of radius proportional to $A^{1/3}$, then this term is

$$f_3(Z,A) = +a_3 \frac{Z^2}{A^{1/3}}$$

(Strictly the numerator should be $Z(Z-1)$, but for large Z it is sufficiently accurate to use Z^2) A similar effect would be present for a charged drop of a classical liquid.

The next term brings in a property specific to nuclei. It is the *asymmetry* term,

$$f_4(Z,A) = +a_4 \frac{(Z - A/2)^2}{A}$$

which accounts for the observed tendency for nuclei to have $Z = N$. (There are no nuclei with very large neutron or proton excesses.) This term is purely quantum mechanical in origin and is due to the Pauli principle. You can see part of the reason for this form from the schematic diagram of Fig.2.5, which shows the energy levels of a nucleus near the highest filled levels (ignoring the spins of the nucleons).

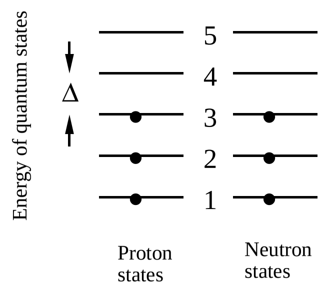

Fig.2.5 Schematic diagram of nuclear energy levels near the highest filled levels. (In an actual nucleus the levels are not equally spaced.)

In this approximation all the energy levels are separated by the same energy Δ. Keeping A fixed and removing a proton from level 3 and adding a neutron to level 4 gives $(N-Z)=2$ and leads to an energy increase of Δ. Repeating this for two nucleons, gives $(N-Z)=4$ and an increase of 4Δ and so on. In general we find that the transfer of $(N-Z)/2$ nucleons decreases the binding energy by an amount $-\Delta(N-Z)^2/4$. Although we have assumed Δ is a constant, in practice it decreases like A^{-1}; hence the final form of the asymmetry term.

In discussing the asymmetry term we should of course have taken account of the spins of the nucleons and the fact that the Pauli principle allows *two* identical nucleons to occupy the same quantum state (corresponding to spin-up and spin-down). If we start with an even number of nucleons and progressively fill states, then the lowest energy will be when both Z and N are even. If on the other hand we have a system where both Z and N are odd, and the highest filled proton state is above the highest filled neutron state, we can increase the binding energy by removing one proton from the nucleus and adding one neutron. If the highest filled proton state is below the highest filled neutron state, then we can produce the same effect by removing a neutron and adding a proton. These observations are summarised in the empirical *pairing* term

$$\begin{aligned} f_5(Z,A) &= -f(A), && \text{if } Z \text{ even, } A-Z=N \text{ even} \\ f_5(Z,A) &= \;\; 0, && \text{if } Z \text{ even, } A-Z=N \text{ odd; or, } Z \text{ odd, } A-Z=N \text{ even} \\ f_5(Z,A) &= +f(A), && \text{if } Z \text{ odd, } A-Z=N \text{ odd} \end{aligned}$$

where the exact form of the function $f(A)$ is found by fitting the data, when the form $f(A) = a_5 A^{-1/2}$ is obtained. The pairing term maximises the binding when both Z and N are even.

To help remember these terms, the notation

$$a_1 = a_v\,, \quad a_2 = a_s\,, \quad a_3 = a_c\,, \quad a_4 = a_a\,, \quad a_5 = a_p$$

is often used. The coefficients are obtained by fitting binding energy data and the result was shown as the solid line in Fig.2.1. Numerical values, in units of MeV/c^2, are:

$$a_v = 15.67\,,\ a_s = 17.23\,,\ a_c = 0.714\,,\ a_a = 93.15\,,\ a_p = 11.2 \qquad (VSCAP)$$

[**Note:** Some books write the asymmetry term as proportional to $(Z-N)^2$ and hence their value for the coefficient a_a will differ by a factor of four from the one above.]

The relative sizes of each of the terms are shown in Fig.2.6. The fit is remarkably good for such a simple form, but not exact of course, and gives accurate values for the binding energies for some 200 stable and many more unstable nuclei. We will use it to analyse the stability of nuclei.

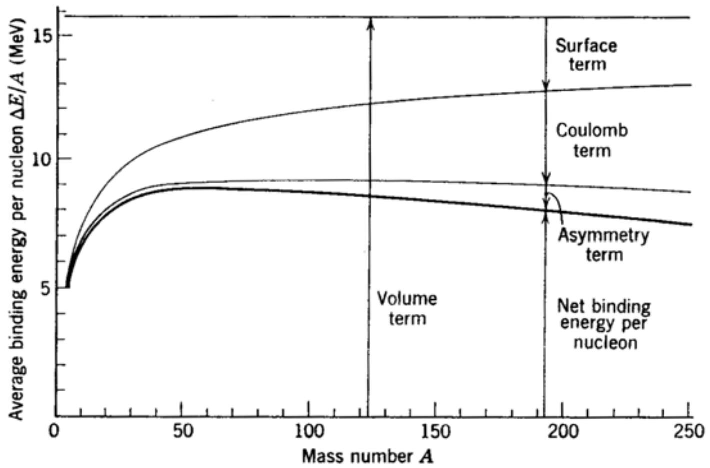

Fig.2.6 Contributions to the binding energy per nucleon as a function of mass number from each term in the semi-empirical mass formula.

2.6 Nuclear stability
Stable nuclei only occur in a very narrow band in the $Z-N$ plane (See Fig.2.7).

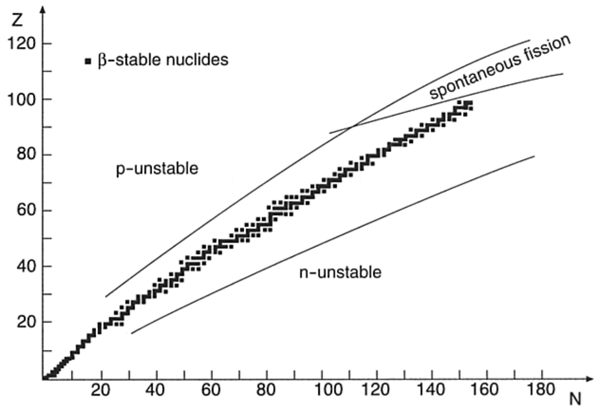

Fig.2.7 The distribution of stable nuclei

All other nuclei are unstable and decay spontaneously in various ways. Isobars with a large surplus of neutrons gain energy by converting a neutron into a proton; conversely a nucleus with a large surplus of protons convert protons to neutrons. These are examples of β-decays, already mentioned. The maximum of the curve of binding energy per nucleon is at around the position of iron (*Fe*) and nickel (*Ni*), which are therefore the most stable nuclides. In heavier nuclei, the binding energy is smaller because of the larger Coulomb repulsion. For still heavier nuclear masses, nuclei become unstable to fission (breakup) and decay spontaneously

into two or more lighter nuclei provided the mass of the parent nucleus is larger than the sum of the masses of the daughter nuclei. Most such nuclei decay via two-body decays and the commonest case is when one of the daughter nuclei is a ^4He nucleus ($^4He \equiv 2p2n$, i.e. $A=4, Z=N=2$), called historically an α-particle. In the rare cases where the two daughters have similar masses, we speak of *spontaneous fission*. This only occurs with a probability greater than that for α-emission for nuclei with $Z \geq 110$. We will briefly discuss each of these possibilities in turn.

2.7 β – decay : phenomenology
By rearranging terms, the semi-empirical mass formula may be written

$$M(Z,A) = \alpha A - \beta Z + \gamma Z^2 + \frac{\delta}{A^{1/2}}$$

where, using our previous notation,

$$\alpha = M_n - a_v + \frac{a_s}{A^{1/3}} + \frac{a_a}{4}$$
$$\beta = a_a + (M_n - M_p - m_e)$$
$$\gamma = \frac{a_a}{A} + \frac{a_c}{A^{1/3}}$$
$$\delta = a_p$$

The electron mass has appeared because from now on I will take the SEMF as applying to *atomic* masses. $M(Z,A)$ is thus a quadratic in Z at fixed A. For odd *A*, the curve is a single parabola. For even *A*, the even-even and odd-odd nuclei lie on two distinct vertically shifted parabolas. This is because of the pairing term. The minimum of the parabolas is at $Z=\beta/2\gamma$. The nucleus with the smallest mass in an isobaric spectrum is stable with respect to β-decay. We will consider the two cases separately using specific values of A to illustrate the main features.

(a) Odd-mass nuclei
The example we take is the case of the $A=101$ isobars, which are shown in Fig.2.8 The parabola minimum is at the isobar $^{101}_{44}Ru$ with $Z=44$. Isobars with more neutrons, such as $^{101}_{42}Mo$ and $^{102}_{43}Te$, decay by converting a neutron to a proton, i.e.

$$n \to p + e^- + \bar{\nu}_e$$

so that

$$^{101}_{42}Mo \to ^{101}_{43}Tc + e^- + \bar{\nu}_e$$

and

$$^{101}_{43}Tc \to ^{101}_{44}Ru + e^- + \bar{\nu}_e$$

This decay sequence is shown in Fig.2.8. Electron emission is energetically possible whenever the mass of the daughter atom $M(Z+1,A)$ is smaller than its isobaric neighbour, i.e.

$$M(Z,A) > M(Z+1,A)$$

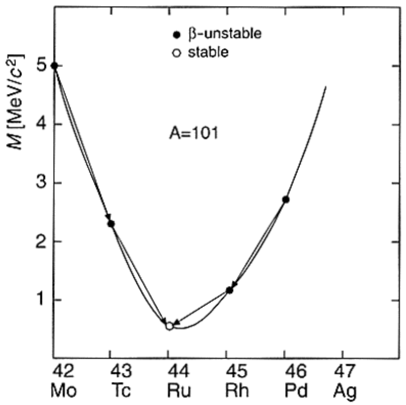

Fig.2.8 Mass parabola of the $A = 101$ isobars. Possible β-decays are shown by arrows. The abscissa is the atomic number Z and the zero point of the mass scale is arbitrary.

Note that we refer here to *atoms*, so that the rest mass of the created electron is automatically taken into account. Isobars with proton excess decay via

$$p \rightarrow n + e^+ + \nu_e$$

i.e. positron emission, which although not possible for a free proton *is* possible in a nucleus because of the binding energy. For example,

$$^{101}_{46}Pd \rightarrow ^{101}_{45}Rh + e^+ + \nu_e$$

and

$$^{101}_{45}Rh \rightarrow ^{101}_{44}Ru + e^+ + \nu_e$$

and once again we arrive at the stable isobar.

Positron emission is energetically possible if

$$M(Z,A) > M(Z-1,A) + 2m_e$$

which takes account of the creation of a positron and the existence of an excess of electrons in the parent atom. It is also theoretically possible for this sequence of decays to occur by *electron capture*. For example, the last step could be

$$e^- + ^{101}_{45}Rh \rightarrow ^{101}_{44}Ru + \nu_e$$

which is a manifestation of the primary reaction

$$e^- + p \rightarrow n + \nu_e$$

Electron capture mainly occurs in heavy nuclei, where the electron orbits are more compact. It is usually the electron in the innermost shell (i.e. the K-shell) that is captured. Capture of

such an electron gives rise to a 'hole' and causes electrons from higher levels to cascade downwards and in so doing emit characteristic X-rays. Electron capture is energetically allowed if

$$M(Z,A) > M(Z-1,A) + \varepsilon$$

where ε is the excitation energy of the atomic shell of the daughter nucleus. The process competes with positron emission.

(b) Even-mass nuclei

Consider as an example the case of $A = 106$ shown in Fig.2.9. The lowest isobar on the lowest curve is $^{106}_{46}Pd$ and is β-stable. The isobar $^{106}_{48}Cd$, also on the lower curve, is also stable since its two odd-odd neighbours both lie above it. In principle, it could decay via double β-decay:

$$^{106}_{48}Cd \rightarrow ^{106}_{46}Pd + 2e^+ + 2\nu_e$$

but this is heavily suppressed to the extent that it is unobservable. Thus, there are two β-stable isobars and this a common situation for *A*-even, although no two neighbouring isobars are known to be stable. Odd-odd nuclei always have at least one more strongly bound, even-even neighbour nucleus in the isobaric spectrum. They are therefore unstable. The only exceptions to this rule are a few very light nuclei

The lifetime of a free nucleon is about 887 s. The free proton is stable and can only 'decay' within a nucleus by utilising the binding energy. Lifetimes of β emitters vary enormously from milliseconds to 10^{16} yrs. They depend very sensitively on the energy E released (the lifetime $\tau \sim 1/E^5$) and on the properties of the nuclei involved, e.g. their spins.

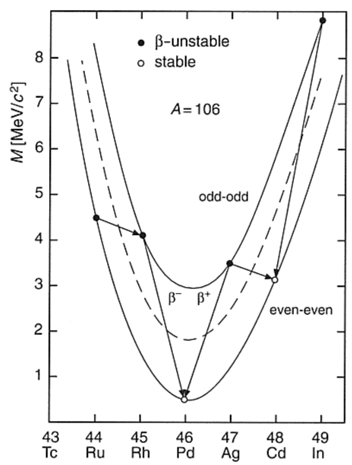

Fig.2.9 Mass parabolas of the $A = 106$ isobars. Possible β-decays are indicated by arrows. The abcissa is the charge number Z and the zero point of the mass scale is arbitrary.

2.8 α-decay

To discuss this, we could return to the semiempirical mass formula (SEMF) and by taking partial derivatives with respect to A and Z find the limits of α-stability, but the result is not very illuminating. To get a very rough idea of the stability criteria, we can write the SEMF in terms of the binding energy *B*. Then α-decay is energetically allowed if

$$B(2,4) > B(Z,A) - B(Z-2,A-4)$$

If we now make the *approximation* that along the line of stability $Z = N$, then there is only one independent variable and if we take this to be *A*,

$$B(2,4) > B(Z,A) - B(Z-2,A-4) \approx 4\frac{dB}{dA}$$

We can then write

$$4\frac{dB}{dA} = 4\left[A\frac{d(B/A)}{dA} + \frac{B}{A}\right]$$

From the plot of B/A, we have $d(B/A)/dA \approx -7.7\text{x}10^{-3}\,\text{MeV}$ for $A \geq 120$ and we also know that $B(2,4) = 28.3\text{MeV}$, so we have

$$28.3 \approx 4\left[B/A - 7.7\text{x}10^{-3}A\right]$$

which is a straight line on the B/A versus *A* plot which cuts the plot at $A = 151$. Above this *A* the inequality is satisfied by most nuclei and α-decay becomes energetically possible.

Lifetimes of α-emitters also span an enormous range, and examples are known from 10 ns to 10^{17} yrs. The origin of this lies in the quantum mechanical phenomenon of *tunelling*, which you met in Second Year quantum mechanics. Individual protons and neutrons have binding energies in nuclei of about 8 MeV, even in heavy nuclei, and so cannot in general escape. However, a bound group of nucleons can sometimes escape because its binding energy increases the total energy available for the process. In practice, the most significant decay process of this type is the emission of an α-particle, which unlike systems of 2 and 3 nucleons is very strongly bound by $7\,\text{MeV/nucleon}$. Fig.2.10 shows the potential energy of an α-particle as a function of *r*, its distance from the centre of the nucleus.

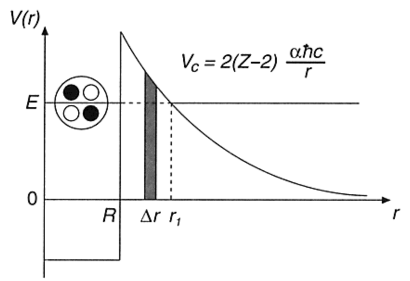

Fig.2.10 Potential energy of an α-particle as a function of its distance from the centre of the nucleus.

Beyond the nuclear force range $r > R$, the α-particle feels only the Coulomb potential

$$V_C(r) = \frac{2(Z-2)\alpha\hbar c}{r}$$

Within the nuclear force range $r < R$, a strong nuclear potential prevails with its strength characterised by the depth of the well. Since the α-particle can escape from the nuclear potential, $E_\alpha > 0$. It is this energy that is released in the decay. Unless E_α is larger than the Coulomb barrier (in which case the decay would be so fast as to be unobservable) the only way the α-particle can escape is by quantum mechanical tunelling through the barrier.

The probability for transmission *T* through a barrier of height *V* and thickness Δr by a particle of mass *m* with energy E_α is given approximately by

$$T \approx e^{-2\kappa\,\Delta r}$$

where $\kappa = \sqrt{2m|E_\alpha - V|}\big/\hbar$. Using this result, we can model the Coulomb barrier as a succession of thin barriers of varying height. The overal tranmission probability is then

$$T = e^{-2G}$$

where the *Gamow factor G* is

$$G = \frac{1}{\hbar}\int_R^{r_1}\sqrt{2m|E_\alpha - V(r)|}\,dr \approx \frac{2\pi\alpha(Z-2)}{\beta}$$

where $\beta = v/c$ and *v* is the velocity of the emitted particle. The probability per unit time λ of the α-particle escaping is proportional to: (a) the probability $w(\alpha)$ of finding the α-particle in the nucleus; (b) the number of collisions of the α-particle with the barrier (this is proportional to $v_0/2R$ where v_0 is the velocity of the α-particle within the nucleus); and (c) the transition probability. Thus

$$\lambda = w(\alpha)\frac{v_0}{2R}e^{-2G}$$

and since

$$G \propto \frac{Z}{\beta} \propto \frac{Z}{\sqrt{E_\alpha}}$$

small differences in E_α have strong effects on the lifetime.

2.9 Fission

Decay via α-emission is an example of *fission*, where a parent nucleus breaks into daughter nuclei. *Spontaneous fission* is the term used to describe the case where the two daughter nuclei are of approximately equal mass. (Precisely equal masses are very unlikely and in the most probable cases the daughter nuclei have mass numbers which differ by about 45.) The binding energy curve shows that this is energetically possible for nuclei with $A > 100$. For example

$$^{238}_{92}U \rightarrow ^{145}_{57}La + ^{90}_{35}Br + 3n$$

with a release of about 156 MeV of energy, which is carried off as kinetic energy of the fission products. Heavy nuclei are neutron-rich and so neccesarily produce neutron-rich decay products, including free neutrons. The fission products are themselves some way from the line of β-stability and will decay by a series of steps. For example, ${}^{145}_{57}La$ decays to the β-stable ${}^{145}_{60}Nd$ by three stages, releasing a further 8.5 MeV of energy, which in this case is carried off by the electrons and neutrinos emitted in β-decay. Although the probability of fission increases with increasing *A*, it is still a very rare process. For example, in ${}^{238}_{92}U$, the transition rate for spontaneous fission is about $3\text{x}10^{-24}\text{s}^{-1}$ compared with about $5\text{x}10^{-18}\text{s}^{-1}$ for α-decay, a branching fraction of $6\text{x}10^{-7}$. Spontaneous emission only becomes dominant in very heavy elements with $A \geq 270$, as we shall now show.

To understand spontaneous fission we can again use the liquid drop model. In the SEMF we have assumed that the drop (i.e. the nucleus) is spherical, because this minimises the surface area. However, if the surface is perturbed for some reason from spherical to prolate, the surface term in the SEMF will increase and the Coulomb term will decrease (assuming the volume remains the same) and the relative sizes of these two changes will determine whether the nucleus is stable against spontaneous fission.

For a fixed volume we can parametrise the deformation by the semi-major and semi-minor axes of the ellipsoid *a* and *b*, respectively. (See Fig.2.11.)

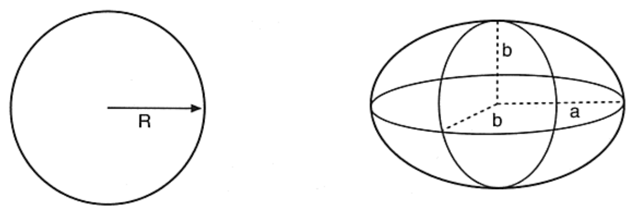

Fig.2.11 Deformation of a heavy nucleus

Thus, we set

$$a = R(1+\varepsilon)\,; \qquad b = R/(1+\varepsilon)^{1/2}$$

which preserves the volume

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi ab^2$$

To find the new surface and Coulomb terms one has to find the expression for the surface of the ellipsoid in terms of *a* and *b* and expand it in a power series in ε. The algebra is unimportant, and I will just quote the results:

$$E_s = a_s A^{2/3}\left(1+\frac{2}{5}\varepsilon^2+...\right) \quad \text{and} \quad E_c = a_c Z^2 A^{-1/3}\left(1-\frac{1}{5}\varepsilon^2+...\right)$$

Hence, the change in the total energy is

$$\Delta E = \frac{\varepsilon^2}{5}\left(2a_s A^{2/3} - a_c Z^2 A^{-1/3}\right)$$

If $\Delta E < 0$, then the deformation is energetically favourable and fission can occur. This happens if

$$\frac{Z^2}{A} \geq \frac{2a_s}{a_c} \approx 48$$

using experimental values for the coefficients a_s and a_c given earlier. This is the case for nuclei with $Z > 114$ and $A \geq 270$.

Spontaneous fission is, like α-decay, a potential barrier problem and this is shown in Fig.2.12. The solid line corresponds to the shape of the potential in the parent nucleus. The height of the barrier determines the probability of spontaneous fission. For very heavy nuclei, the shape of the potential corresponds to the dashed line and the slightest deformation will induce fission. For very heavy nuclei ($Z \geq 92$) the fission barriers are only about 6 MeV. In principle, the nucleus could fission by tunelling through the barrier, but the fragments are large, the Gamow factor is very small and thus the probability for this to happen is extremely small.

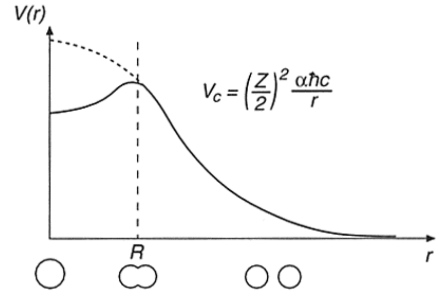

Fig.2.12 Potential energy during different stages of a fission reaction.

Another possibility is to supply this energy by a flow of neutrons. A neutron can get very close to the nucleus and be captured by the strong nuclear attraction. The parent nucleus may then be excited to a state above the fission barrier and therefore split up. This process is called *induced fission*. Neutron capture by a nucleus with an odd neutron number releases not just some binding energy, but also a pairing energy. This small extra contribution makes a crucial difference to nuclear fission properties. We will see that very low-energy ('thermal') neutrons can induce fission in ${}^{235}U$, whereas only higher energy ('fast') neutrons induce fission in ${}^{238}U$. This is because ${}^{235}U$ is an even-odd nucleus and ${}^{238}U$ is even-even. Therefore, the ground state of ${}^{235}U$ will lie higher (less tightly bound) in the potential well of its fragments than that of ${}^{238}U$. Hence to induce fission, a smaller energy will be needed for ${}^{235}U$ than for ${}^{238}U$.

We consider this qualitatively as follows. The capture of a neutron by ${}^{235}U$ changes an even-odd nucleus to a more tightly bound even-even (compound) nucleus of ${}^{236}U$ and releases the binding energy of the last neutron. In ${}^{235}U$ this is 6.5 MeV. The energy needed to induce fission (i.e. the *activation energy*) is calculated to be about 5 MeV for ^{236}U and thus neutron capture releases sufficient energy to fission the nucleus. The kinetic energy of the incident neutron is irrelevant and thermal neutrons can induce fission in ^{235}U. In contrast, neutron capture in ^{238}U changes it from an even-even nucleus to an even-odd nucleus, i.e. changes a tightly bound nucleus to a less tightly bound one. The energy released (the binding energy of the last neutron) is about 4.8 MeV in ^{239}U and is less than the 6.5 MeV required for fission. For this reason fast neutrons with an energy of at least this difference are required.

2.3 $\boldsymbol{\gamma}$ – decays

When a heavy nucleus disintegrates by either α or β decay, or by fission, the daughter nucleus is often left in an excited state. If this state does not itself also disintegrate, it will de-excite, usually by emitting a high-energy photon, called in this context a gamma-ray (γ). The energy of these photons is determined by the average energy level spacings in nuclei and ranges fom a few to several MeV. Because γ-decay is an electromagnetic process, we would expect the typical lifetime of an excited state to be $\sim 10^{-16}$ s. In practice, we have seen that lifetimes are very sensitive to the amount of energy released in the decay and in the nuclear case other factors are also very important, particulrly the quantity of angular momentum carried off by the photon. Typical lifetimes of nuclear levels are about $\sim 10^{-12}$ s.
The role of angular momentum in γ-decays is crucial. If the initial (excited) state has a total spin $\mathbf{J}_i$ and the final nucleus has a total spin $\mathbf{J}_f$, then the total angular momentum $\mathbf{L}$ of the emitted photon is given by

$$\mathbf{L} = \mathbf{J}_i - \mathbf{J}_f$$

with

$$J_i + J_f \geq L \geq \left| J_i - J_f \right|$$

and

$$m_i = M + m_f$$

where the latter are the corresponding magnetic quantum numbers. There is a further constraint because in electromagnetic processes parity is conserved. This is complicated because both the initial and final nuclear level will have an intrinsic parity as does the photon and in addition there is a parity associated with the angular momentum carried off by the photon.

The quantity of angular momentum carried off in the decay (in units of $\hbar$) is called the *multipolarity*. It determines the angular distribution of the radiation and in the semiclassical theory of radiation is used to classify the type of radiation emitted. For example, for the transition $\mathbf{J}_i = \mathbf{1} \to \mathbf{J}_f = \mathbf{0}$, the photon must have $L = 1$. This is called an electric dipole (E1) transition and is accompanied by a change of parity between initial and final nuclear states. (Overall parity conservation is achieved by including the parity of the photon.) For the transition $\mathbf{J}_i = \mathbf{2} \to \mathbf{J}_f = \mathbf{1}$, L could be 1, 2 or 3, but will almost certainly be 1, because the lowest value is normally highly preferred. We will not pursue this further, because to do so requires a knowledge of radiation in quantum theory, which you have yet to meet.