1. BASIC IDEAS

1.1 History

Although this course will not follow a strict historical development, I will start with a few brief remarks about the origins of nuclear and particle physics.

In the 19th century, atoms were considered indivisible and were the elementary particles of their day. This view changed in 1897 with Thomson's discovery of the electron (e^-), followed 14 years later by the classic experiments of Rutherford, in which a beam of electrons was fired into a thin gold foil. You will recall from your atomic physics lectures that from the angular distribution of the scattered electrons, Rutherford deduced that atoms consisted of a tiny positively charged central *nucleus*, orbited by electrons. This led to the Bohr model of the atom, which is familiar to you from atomic and quantum physics lectures. Later experiments further shattered the 19th century view by showing that the nucleus was itself composite, in fact a bound state of two particles: the *proton* (*p)* with electric charge +*e* (where *e* is the magnitude of the charge on the electron) and the electrically neutral *neutron (n).* (Protons and neutrons are collectively called *nucleons*.) Thus, by the early 1930s, atoms had been replaced as elementary particles by a larger group of smaller entities: electrons, protons and neutrons. To these we should add two electrically neutral particles: the *photon* (γ) (postulated in 1900 by Planck to explain black-body radiation), and the *neutrino* (ν) (postulated by Fermi in 1930 to explain the apparent non-conservation of energy observed in the decay products of some unstable nuclei, so-called $\beta-$*decays*, which we discuss later in this course). All this changed again in the late 1960s by a series of experiments analogous to those of Rutherford, where high-energy beams of electrons and neutrinos were scattered from nucleons. Analysis of the angular distributions of the scattered particles showed that the nucleons were themselves bound states of three point-like charged entities, which we now call *quarks* (q), with the unusual property of having fractional electrical charges – in practice $-e/3$ and $2e/3$. This is essentially the picture today, where elementary particles are considered to be a small number of entities, including quarks, the electron, neutrinos, the photon and a few others we shall meet, and nuclei are bound states of protons and neutrons.

1.2 The Standard Model

The best theory of elementary particles we have today is called the *standard model*. This aims to explain all the phenomena of particle physics, except gravity, in terms of the properties and interactions of a small number of *elementary (*or *fundamental) particles*, which we now define as being point-like, without internal structure or excited states. Such a particle is characterised by, amongst other things, its mass, its electric charge and its *spin*. You will recall from quantum mechanics lectures that spin is a permanent angular momentum possessed by particles in quantum theory, even when they are at rest, and that the maximum value of the spin angular momentum about any axis is $sh/2\pi$, where h is Planck's constant and s is the *spin quantum number*, or *spin* for short. It has a fixed value for particles of any given type (for example $s=\frac{1}{2}$ for electrons) and general quantum mechanical principles restrict the possible values of s to be $0, \frac{1}{2}, 1, \frac{3}{2}, ...$ Particles with half-integer spin are called *fermions* and those with integer spin are called *bosons*. In the standard model there are three families of elementary particles: two spin-1/2 families of fermions called *leptons* and *quarks*; and one family of spin-0 *gauge bosons*. In addition, at least one other spin-0 particle, called the *Higgs boson*, is postulated to explain the origin of mass within the theory. The latter will be discussed in the 4th Yr lectures on elementary particle physics, although I will say a few brief words about it later in the course.

The most familiar example of a lepton is the electron. This is bound in atoms by the electromagnetic interaction, one of the three forces of nature. (There is also a fourth force – gravity – but this is so small for elementary particles that I will neglect it.) Another lepton is the neutrino, which was mentioned earlier as a decay product in β-decays. (Strictly this particle should be called the *electron neutrino*, written ν_e, because it is always produced in association with an electron – more about why this happens later in the course.) The force responsible for beta decay is an example of a second fundamental force, the *weak interaction*. Finally, there is the third force, the *strong interaction*, which, for example, binds quarks in nucleons.

In classical physics the *electromagnetic interaction* is propagated by electromagnetic waves, which are continuously emitted and absorbed. While this is an adequate description at long distances, at short distances the quantum nature of the interaction must be taken into account. In quantum theory, the interaction is transmitted discontinuously by the exchange of photons, which are spin-1 bosons. Photons are referred to as the *gauge bosons*, or "*force carriers*", of the electromagnetic interaction. The weak and strong interactions are also mediated by the exchange of spin-1 gauge bosons. For the weak interaction these are the W^+, W^- and Z^0 *bosons* (the superscript denotes the electric charge) with masses about 80-90 times the mass of the proton. For the strong interaction, the force carriers are called *gluons*. There are eight gluons, all of which have zero mass and are electrically neutral. Note that I have used the word 'electric' when talking about 'charge'. This is because the weak and strong interactions also have associated 'charges' which determine the strengths of the interactions, just as the electric charge determines the strength of the electromagnetic interaction. I will say more about this later in the course.

In addition to the elementary particles of the standard model, there are other important particles we will be studying. These are the *hadrons*, the bound states of quarks. Nucleons are examples of hadrons, but there are several hundred more, not including nuclei, most of which are unstable and decay by one of the three interactions. A very common example is the *pion*, which exists in three electrical charge states, written (π^+,π^0,π^-). Hadrons are important because free quarks are unobservable in nature (we will discuss the reason for this later) and so to deduce their properties we are forced to study hadrons. (The analogy would be having to deduce the properties of protons and neutrons by studying the properties of nuclei.) Since nucleons are bound states of quarks and nuclei are bound states of nucleons, the properties of nuclei should in principle be deducible from the properties of quarks and their interactions, i.e. from the standard model. In practice, however, this is far beyond present calculational technique (an analogy might be to deduce the behaviour of the human body solely from its biochemical reactions) and often nuclear and particle physics are treated as two almost separate subjects.

1.3 Relativity and antiparticles

Elementary particle physics is often called *high-energy* physics. One reason for this is that if we wish to produce new particles in a collision between two other particles, then because of the relativistic mass-energy relation $E=mc^2$, high energies are needed, at least as great as the rest masses of the particles produced. The second reason is that to explore the structure of a particle requires a probe whose wavelength λ is at least as small as the structure to be explored. By the de Broglie relation $\lambda = h/p$, this implies that the momentum p of the probing particle, and hence its energy, must be large. For example, to explore the internal structure of the proton using electrons requires wavelengths that are much smaller than the radius of the proton, which is roughly $10^{-15}m$. This in turn requires electron energies that are greater than 10^3 times the rest energy of the electron, implying electron velocities very close to the speed of light. Hence any explanation of the phenomena of elementary particle physics must take account of the requirements of the theory of special relativity, in addition to those of quantum theory.

Constructing a quantum theory of elementary particles which is consistent with special relativity leads to the conclusion that for each particle of nature, whether it is an elementary particle or a hadron, there must exist an associated particle, called an *antiparticle*, with the same mass as the corresponding particle. If the particle is electrically charged, then the antiparticle will have the opposite charge. Experimental evidence confirms this important theoretical prediction. If we write the particle as P, then the antiparticle is in general written with a bar over it, i.e. $\overline{P}$. For example, associated with every quark, q, is an antiquark, $\bar{q}$. However, for very common particles the bar is often omitted. Thus, for example, the negatively charged electron e^- has an antiparticle e^+, called the *positron*. In this case the superscript denoting the charge makes explicit the fact that the antiparticle has the opposite electric charge to that of its associated particle. Electric charge is just one example of a *quantum number* (spin, introduced earlier, is another) that characterises a particle, whether it is elementary or composite (i.e. a hadron). Many quantum numbers differ in sign for particle and antiparticle, and electric charge is an example of this. We will meet others later. When brought together, particle-antiparticle pairs can annihilate each other, releasing their combined rest energy $2mc^2$ as photons or other particles. Finally, we note that there is symmetry between particles and antiparticles, and it is a convention which is which; for example, we could call the positron the particle, and the electron the antiparticle. That we do not do so merely reflects the fact that the matter around us contains electrons rather than positrons, rather than the other way round.

1.4 Particle reactions

Reactions involving elementary particles and/or hadrons are summarised by equations by analogy with chemical reactions, in which the different particles are represented by symbols, which sometimes, but not always, have a superscript to denote their electric charge. In the reaction

$$\nu_e + n \to e^- + p$$

for example, an electron neutrino ν_e collides with a neutron n to produce an electron e^- and a proton *p;* while the equation

$$e^- + p \to e^- + p$$

represents an electron and proton interacting to give the same particles in the final state, but travelling in different directions. This latter type of reaction, in which the particles remain unchanged, is called *elastic scattering*, while the first reaction is an example of *inelastic scattering*. Collisions between given initial particles do not always lead to the same final state, but can lead to different final states with different probabilities. For example, an electron-positron collision can give rise to elastic scattering

$$e^+ + e^- \to e^+ + e^-$$

or annihilation, an inelastic reaction, to give either two or three photons in the final state

$$e^+ + e^- \to \gamma + \gamma \quad \text{or} \quad e^+ + e^- \to \gamma + \gamma + \gamma$$

Finally, some particles are unstable and spontaneously decay to other, lighter particles. An example of this is the neutron, which decays by the β–decay reaction

$$n \to p + e^- + \bar{\nu}_e$$

with a mean lifetime of about 900 seconds. (The reason that this involves an antineutrino rather than a neutrino will become clear presently.) Many nuclei also decay via the β–decay reaction. Thus, denoting a nucleus with Z protons and N nucleons as (Z, N), we have, for example

$$(Z,N) \to (Z+1,N) + e^- + \bar{\nu}_e$$

This reaction is, in effect, the decay of a neutron bound in a nucleus.

1.5 Feynman diagrams

Particle reactions, like those above, are brought about by the fundamental forces between the elementary particles involved. A convenient way of illustrating this is to use *Feynman diagrams*. There are mathematical techniques associated with these, which enable them to be used to calculate the quantum mechanical probabilities for given reactions to occur, but in these lectures they will only be used as a convenient pictorial description of reaction mechanisms. We first illustrate them for the case of electromagnetic reactions, which arise from the emission and/or absorption of photons. For example, the dominant interaction between two electrons is due to the exchange of a single photon, which is emitted by one electron and absorbed by the other. This mechanism, which gives rise to the familiar Coulomb interaction at large distances, is illustrated in the Feynman diagram Fig.1.1a.

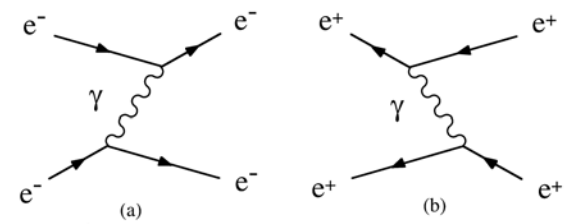

Fig.1.1 One-photon exchange in (a) $e^+ + e^- \to e^- + e^-$ and (b) $e^+ + e^+ \to e^+ + e^+$

In such diagrams, by convention, the initial particles are shown on the left and the final particles to the right. Spin-1/2 fermions (such as the electron) are drawn as solid lines and photons are drawn as wiggly lines. The arrow heads pointing to the right indicate that the solid lines represent electrons. In the case of photon exchange between two positrons, which is shown in Fig.1.1b, the arrowheads on the antiparticle (positron) lines are conventionally shown as pointing to the left.

The dominant contribution to the annihilation reaction $e^+e^- \to \gamma\gamma$ is shown in Fig.1.2. The positron emits a photon and then annihilates with an electron to produce the second photon. (You could also draw another diagram where the electron emits the photon before annihilating with the positron to produce the second photon.) In interpreting these diagrams, it is important to remember that the direction of the arrows on fermion lines do not indicate their direction of motion, but merely whether the fermions are particles or antiparticles; and that the initial particles are always to the left and the final particles to the right.

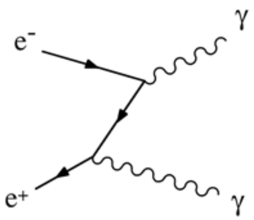

Fig.1.2 The reaction $e^+ + e^- \to \gamma + \gamma$

A feature of the above diagrams is that they are constructed from a single fundamental three-line vertex. This is characteristic of electromagnetic processes. Each vertex has a line corresponding to a single photon being emitted or absorbed; while one fermion line has the arrow pointing toward the vertex and the other away, guaranteeing charge conservation at the vertex, which is one of the rules of Feynman diagrams (c.f. Kirchhoff's laws in electromagnetism.) For example, a vertex like Fig.1.3 would correspond to a process in which an electron emitted a photon and turned into a positron. This would violate charge conservation and is therefore forbidden.

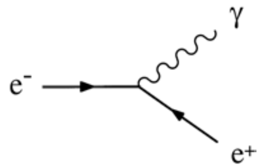

Fig.1.3 The forbidden vertex $e^- \to e^+ + \gamma$

Feynman diagrams can also be used to describe the fundamental weak and strong interactions. This is illustrated by Fig.1.4, which shows the dominant contributions to the elastic scattering reaction $\nu_e + e^- \to \nu_e + e^-$ and Fig.1.5, which shows the exchange of a single gluon (represented by a coiled line) between two quarks.

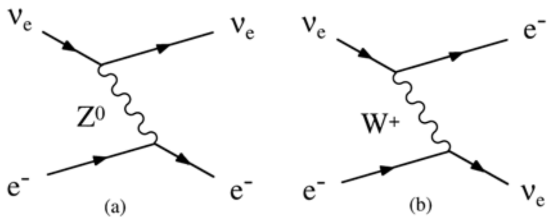

Fig.1.4 W and Z^0 exchange contributions to the reaction $\nu_e + e^- \to \nu_e + e^-$

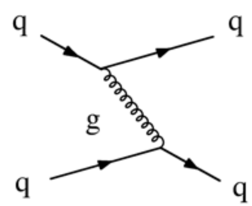

Fig.1.5 Single-gluon exchange in the reaction $q + q \to q + q$

1.6 Particle exchange – range of forces

At each vertex of a Feynman diagram, charge is conserved. We will see later that, depending on the nature of the interaction (strong, weak or electromagnetic), other quantum numbers are also conserved. However, it is easy to show that energy and momentum *cannot* be conserved simultaneously.

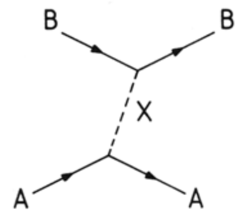

Fig.1.6 Exchange of a particle *X* in the reaction $A + B \to A + B$

Consider a general case of a reaction $A + B \to A + B$ mediated by the exchange of a particle *X*, as shown in the Feynman diagram of Fig.1.6. In the rest frame of the incident particle A, the lower vertex represents the process,

$$A(M_A c^2, \mathbf{0}) \to A(E_A, \mathbf{p_A} c) + X(E_X, -\mathbf{p_A} c)$$

where E_A is the *total* energy of particle *A*. Thus, if we denote by P_i the 4-vector for particle *A*, then

$$P_A = (E_A, \mathbf{p}_A c)$$

and for two particles *A* and *B*, the 4-vector product is

$$P_A P_B = E_A E_B - \mathbf{p}_A \mathbf{p}_B c^2$$

so that

$$P_A{}^2 = E_A{}^2 - \mathbf{p}_A{}^2 c^2 = M_A{}^2 c^4$$

Applying this to the diagram, gives $E_A = (p^2c^2 + M_A{}^2c^4)^{1/2}$, $E_X = (p^2c^2 + M_X{}^2c^4)^{1/2}$, $p = |\mathbf{p}|$ and momentum conservation has been imposed. This is called a *virtual* process because *X* does not appear as a real particle in the final state. The energy difference between the final and initial states is given by

$$\Delta E = E_X + E_A - M_A c^2 \to 2pc\,, \qquad p \to \infty$$
$$\to M_X c^2\,, \qquad p \to 0$$

Thus $\Delta E \geq M_X c^2$ for all p, i.e. energy is not conserved. However, by the Heisenberg uncertainty principle, such an energy violation is allowed, but only for a time $\tau \approx \hbar/\Delta E$, where $\hbar \equiv h/2\pi$, so we immediately obtain

$$r \approx R \equiv \hbar / M_X c$$

as the maximum distance over which X can propagate before being absorbed by particle B. This maximum distance is called the *range* of the interaction.

The electromagnetic interaction has an infinite range because the exchanged particle is a massless photon. In contrast, the weak interaction is associated with the exchange of very heavy particles – the W and Z bosons. This leads to ranges that are of order $R_{W,Z} \approx 2\text{x}10^{-18}$m. In many applications, this range is very small compared to the de Broglie wavelengths of all the particles involved. The weak interaction can then be approximated by a zero range or point interaction, corresponding to the limit as shown in Fig.1.7.

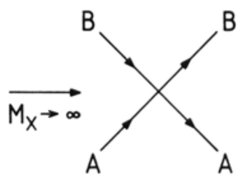

Fig.1.7 Zero-range interaction in the limit $M_X \to \infty$

The fundamental strong interaction has infinite range because, like the photon, gluons have zero mass. However, hadrons experience a strong *short-range* interaction, which in the case of two nucleons, for example, has a range of about 10^{-15}m, corresponding to the exchange of a particle with an effective mass of about 1/7 of the mass of the proton. This should properly be called a "residual", or nuclear, strong interaction. It is a complicated effect due to the interactions between the charge distributions within the two hadrons. Two neutral atoms also experience an interaction (van der Waals force), which although it has its origins in the fundamental Coulomb forces, is of much shorter range. Although an analogous mechanism is not in fact responsible for the nuclear strong interaction, it does illustrate that the force between distributions of particles can be much more complicated than the simpler forces between their components. We will return to the nature of the nuclear force later in this course.

1.7 Yukawa potential

In the limit that M_A becomes large, we can regard B as being scattered by a static potential of which A is the source. This potential will in general be spin dependent, but its main features can be obtained by neglecting spin and considering X to be a spin-0 boson, in which case it will obey the *Klein-Gordon equation.*

$$-\hbar^2 \frac{\partial^2 \phi(\mathbf{x},t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \phi(\mathbf{x},t) + M_X{}^2 c^4 \phi(\mathbf{x},t)$$

This is a relativistic equation, which is derived by starting from the relativistic mass-energy relation $E^2 = p^2c^2 + M^2c^4$ and using the usual quantum mechanical operator substitutions

$$\mathbf{p} = -i\hbar \frac{\partial}{\partial \mathbf{x}} \qquad \text{and} \qquad E = i\hbar \frac{\partial}{\partial t}$$

The static solution of the Klein-Gordon equation satisfies

$$\nabla^2 \phi(\mathbf{x}) = \frac{M_X{}^2 c^2}{\hbar^2} \phi(\mathbf{x})$$

where we interpret $\phi(\mathbf{x})$ as a static potential. For $M_X = 0$ this equation is the same as that obeyed by the electrostatic potential, and for a charge $-e$ interacting with a point charge $+e$ at the origin, the appropriate solution is the Coulomb potential

$$V(r) = -e\phi(r) = -\frac{e^2}{4\pi\varepsilon_0} \frac{1}{r}$$

where $r = |\mathbf{x}|$ and ε_0 is the dielectric constant. The corresponding solution in the case where $M_X{}^2 \neq 0$ is

$$V(r) = -\frac{g^2}{4\pi} \frac{e^{-r/R}}{r}$$

where R is the range defined earlier and g, the so-called *coupling constant*, is a parameter associated with each vertex of a Feynman diagram and represents the basic strength of the interaction. (Although we call g a constant, in general it will have a dependence on the momentum carried by the exchanged particle. We ignore this in what follows.) For simplicity we have assumed equal coupling strengths for the coupling of particle X to the particles A and B.

This form of potential is called a *Yukawa potential*, after Hideki Yukawa who first introduced the idea of forces due to massive particle exchange in 1935. For $M_X = 0$, it reduces to the familiar Coulomb form, while for very large masses the interaction is approximately point-like. It is conventional to introduce a dimensionless parameter α_X by

$$\alpha_X = \frac{g^2}{4\pi\hbar c}$$

which characterises the strength of the interaction at short distances $r \leq R$. For the electromagnetic interaction this is the *fine structure constant*

$$\alpha \equiv e^2 / 4\pi\varepsilon_0 \hbar c \approx 1/137$$

although recall the earlier remark that in general α_X will have a dependence on the momentum carried by particle X. In the case of the electromagnetic force this dependence is relatively weak.

1.8 The scattering amplitude

We have mentioned earlier that Feynman diagrams can be turned into probabilities for a process by a complicated set of mathematical rules. We will not pursue this in detail in this course, but I will show in principle the relation to *observables*, i.e. things that can be measured, concentrating on the case of a two-body scattering reaction. The intermediate step is the *amplitude f*, the modulus squared of which is directly related to the probability of the process occurring. It is also called the *invariant amplitude* because it should be the same in all inertial frames of reference. To get some idea of the structure of *f*, we will use non-relativistic quantum mechanics and assume that the coupling constant *g* is small compared to $\sqrt{4\pi\hbar c}$ so that the interaction is a small perturbation on the free particle solution, which we take as plane waves. In lowest order perturbation theory (i.e. in an expansion of the amplitude in powers of g^2, we keep only the first term), the amplitude for a particle to be scattered from an initial momentum $\mathbf{q}_i$ to a final moment $\mathbf{q}_f$ by a potential $V(\mathbf{x})$ is proportional to

$$f(\mathbf{q}) = \int d^3\mathbf{x}\, V(\mathbf{x}) \exp[i\mathbf{q}.\mathbf{x}/\hbar]$$

i.e. the Fourier transform of the potential, where $\mathbf{q} = \mathbf{q}_i - \mathbf{q}_f$ is the momentum transfer from initial to final states. (If you have not seen this before, you can find a derivation in many quantum mechanics books, e.g. F Mandel, *Quantum Mechanics*, sec 10.2.2.)

The integration may be done by using the substitutions

$$\mathbf{q}.\mathbf{x} = |\mathbf{q}|\, r\cos\theta$$

and

$$d^3\mathbf{x} = r^2 \sin\theta\, d\theta\, dr\, d\phi$$

where $r = |\mathbf{x}|$. For the Yukawa potential, this gives

$$f(\mathbf{q}) = \frac{-g^2\hbar^2}{|\mathbf{q}|^2 + M_X{}^2c^2}$$

This amplitude corresponds to the exchange of a single particle, as shown for example in Figs.1.2 and 1.4. The structure of the amplitude is a numerator, which is proportional to the product of the couplings at the two vertices (or equivalently α_X in this case), and a denominator which depends on the mass of the exchanged particle and the momentum transfer squared. The denominator is called the *propagator* for particle *X*. In a relativistic calculation, the term $|\mathbf{q}|^2$ becomes q^2, where *q* is the *four-momentum* transfer.

All the above is for the exchange of a single particle. It is also possible to drawn more complicated Feynman diagrams, that correspond to the exchange of more than one particle. An example of such a diagram for elastic e^-e^- scattering is shown in Fig.1.8

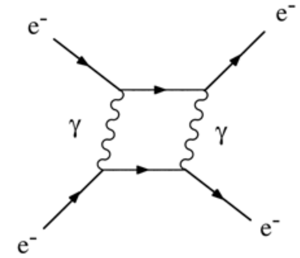

Fig.1.8 Two-photon exchange in the reaction $e^- + e^- \to e^- + e^-$

The number of vertices in any diagram is called the *order n*, and when the probability associated with any given Feynman diagram is calculated, it always contains a factor of α^n. The probability associated with the single-photon exchange diagrams of Fig.1.1 thus contain a factor of α^2 and the contribution from two-photon exchange is of order α^4. The latter is very small compared to the contribution from single-photon exchange because α is a small number. This is again a general feature of electromagnetic interactions. Because the fine structure constant is very small, only the lowest-order diagrams which contribute to a given process need be taken into account, and more complicated higher-order diagrams with more vertices can to a good approximation be ignored in most applications.

1.9 Cross-sections

The next step is to relate the amplitude to measurables. For scattering reactions the appropriate observable is the *cross-section*. In a typical scattering experiment, a beam of particles is allowed to hit a target and the rates of production of various particles in the final state are counted. (We will discuss more about the practical aspects of experiments later in the lectures.) The rates will be proportional to: (a) the number *N* of particles in the target illuminated by the beam; and (b) the rate per unit area at which beam particles cross a small surface placed in the beam at rest with respect to the target and perpendicular to the beam direction. The latter is called the *flux* and is given by

$$J = n_b v_i$$

where n_b is the density of particles in the beam and v_i their velocity in the rest frame of the target. Hence the rate W_r at which a specific reaction occurs in a particular experiment can be written in the form

$$W_r = JN\sigma_r$$

where σ_r is called the *cross-section* for reaction *r*. The product *JN* is called the *luminosity L*, i.e.

$$L \equiv J\, N$$

and contains all the dependencies on the densities and geometries of the beam and target. The cross-section is independent of these factors. You can see from the above equations that the cross-section has the dimensions of area; and the rate per target particle $J\sigma_r$ at which the reaction occurs is equal to the rate at which beam particles would hit a surface of area σ, placed in the beam at rest with respect to the target and perpendicular to the beam direction. Since the area of such a surface is unchanged by a Lorentz transformation in the beam direction, the cross-section is the same in all inertial frames of reference; i.e. it is a Lorentz invariant.

The quantity σ_r is better named as the *partial cross-section*, because it is the cross-section for a particular reaction *r*. The *total cross-section* σ is defined by

$$\sigma \equiv \sum_r \sigma_r$$

Another useful quantity is the *differential cross-section,* $d\sigma_r(\theta,\phi)/d\Omega$, which is defined by

$$dW_r \equiv JN \frac{d\sigma_r(\theta,\phi)}{d\Omega} d\Omega$$

where dW_r is the measured rate for the particles to be emitted into an element of solid angle $d\Omega = d\cos\theta\, d\phi$ in the direction (θ,ϕ). The partial cross-section may be obtained by integrating over angles, i.e.

$$\sigma_r = \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta \frac{d\sigma_r(\theta,\phi)}{d\Omega}$$

The final step is to write these formulas in terms of the scattering amplitude $f(q)$ we defined earlier as appropriate for describing the scattering of a non-relativistic spinless particle from a potential. To do this it is convenient to consider a single beam particle interacting with a single target particle and to confine the whole system in a cube of arbitrary volume *V*, which cancels in the calculation and which we will take to be a cube of side *L*. The incident flux is then given by

$$J = n_b v_i = v_i / V$$

and since the number of target particles is $N = 1$, the differential rate is

$$dW_r = \frac{v_i}{V} \frac{d\sigma_r(\theta,\phi)}{d\Omega} d\Omega$$

In quantum mechanics, provide the interaction is not too strong, the transition rate for any process is given in perturbation theory by

$$dW_r = \frac{2\pi}{\hbar} \left| \int d^3\mathbf{x}\, \psi_\mathbf{f}^* V(\mathbf{x}) \psi_\mathbf{i} \right|^2 \rho(E_f)$$

This equation is a form of *Fermi's Second Golden Rule,* which you will meet in quantum mechanics. The term $\rho(E_f)$ is the *density-of-states factor* and we take the initial and final state wavefunctions to be plane waves:

$$\psi_i = \frac{1}{\sqrt{V}} \exp\left[i\mathbf{q_i}.\mathbf{x}/\hbar\right], \qquad \psi_f = \frac{1}{\sqrt{V}} \exp\left[i\mathbf{q_f}.\mathbf{x}/\hbar\right]$$

where the final momentum $\mathbf{q}_f$ lies within a small solid angle $d\Omega$ located in the direction (θ,ϕ). Thus,

$$dW_r = \frac{2\pi}{\hbar V^2} |f(\mathbf{q})|^2 \rho(E_f)$$

where $f(\mathbf{q})$ is the scattering amplitude defined previously. The density of states $\rho(E_f)$ is calculated by setting $\rho(E)dE$ equal to the number of possible quantum states of the final-state particles which have a total energy between E and $E + dE$. It is found by firstly evaluating $\rho(q)$ where $\rho(q)dq$ is the number of possible final states with $q = |\mathbf{q}|$ lying between q and $q + dq$ and then changing variable using

$$\rho(q) \frac{dq}{dE} dE = \rho(E) dE$$

The possible values of the momentum q are restricted by the boundary conditions to be

$$q_x = \left(\frac{2\pi\hbar}{L}\right) n_x, \quad q_y = \left(\frac{2\pi\hbar}{L}\right) n_y, \quad q_z = \left(\frac{2\pi\hbar}{L}\right) n_z$$

where n_x etc are integers. Hence the number of final states with momentum lying in the momentum space volume

$$d^3\mathbf{q} = q^2 dq\, d\Omega$$

corresponding to momenta pointing into the solid angle $d\Omega$ with momentum between q and $q + dq$ is given by

$$\rho(q)dq = \left(\frac{L}{2\pi\hbar}\right)^3 d^3\mathbf{q} = \frac{V}{(2\pi\hbar)^3} q^2 dq\, d\Omega$$

The derivative

$$\frac{dq}{dE} = \frac{1}{v}$$

and so

$$\rho(E_f) = \frac{V}{(2\pi\hbar)^3} \frac{q_f^{\,2}}{v_f} d\Omega$$

If we use this in the expression for dW_r, we have

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2\hbar^4} \frac{q_f^{\,2}}{v_i v_f} |f(\mathbf{q})|^2$$

This is the final result and is actually also true for the general two-body relativistic scattering process

$$A(\mathbf{q}_i) + B(-\mathbf{q}_i) \to A(\mathbf{q}_f) + B(-\mathbf{q}_f)$$

although the precise form of the external factors depend on the spins of the particles.

1.10 Unstable particles

In the case of an unstable state, the observable of interest is its lifetime at rest τ, or equivalently its natural decay width, given by $\Gamma = \hbar/\tau$ which is a measure of the rate of the decay reaction. In general, an initial unstable state will decay to several final states and in this case we define Γ_f as the *partial width* for channel *f* and

$$\Gamma = \sum_f \Gamma_f$$

as the *total decay width*, while

$$B_f \equiv \Gamma_f / \Gamma$$

is defined as the *branching ratio* for decay to channel *f*.

The energy decay distribution of an unstable state has the characteristic *Breit-Wigner* form

$$P_f(W) \propto \frac{\Gamma_f}{(W-M)^2 c^4 + \Gamma^2/4}$$

where *M* is the mass of the decaying state and *W* is the invariant mass of the decay products. (The overall factor depends on the spins of the particles involved.). This form arises from a state that decays exponentially with time, although a proper proof of this is quite lengthy. (See e.g. Appendix B of Martin and Shaw, *Particle Physics.*) A plot of this formula is shown in Fig.1.9. This is the same formula that describes the widths of atomic spectral lines.

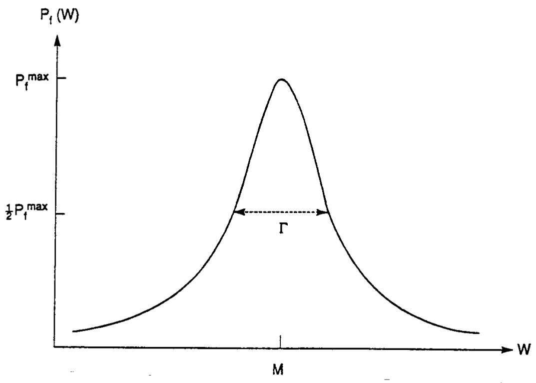

Fig.1.9 The Breit-Wigner formula

If an unstable state is produced in a scattering reaction, then the cross-section for that reaction will show an enhancement described by the Breit-Wigner formula. In this case we say we have produced a *resonance state*. In the vicinity of a resonance of mass *M*, the cross-section for the reaction $i \to f$ will have the form

$$\sigma_{if} \propto \frac{\Gamma_i \Gamma_f}{(E - Mc^2)^2 + \Gamma^2/4}$$

where *E* is the total energy of the system. Again, the form of the overall constant will depend on the spins of the particles involved.

1.11 Units: length, mass and energy

Most branches of science introduce special units that are convenient for their own purposes. Nuclear and particle physics are no exceptions Distances tend to be measured in femtometres or, equivalently *fermis*, with $1\text{fm} \equiv 10^{-15}\text{m}$. In these units, the radius of the proton is about 0.8 fm. The range of the nuclear force between protons and neutrons is of order 1–2 fm, while the range of the weak force is of order 10^{-3}fm. For comparison, the radii of atoms are of order 10^5fm. A common unit for area is the *barn* defined by $1\text{b} = 10^{-28}\text{m}^2$. For example, the total cross-section for *pp* scattering (a strong interaction) is a few tens of millibarns (mb) (nuclear cross-sections are very much larger), whereas the same quantity for *νp* scattering (a weak interaction) is a few tens of femtobarns (fb), depending on the energies involved.

Energies are invariably specified in terms of the electron volt, eV, defined as the energy required to raise the electric potential of an electron or proton by one volt. In terms of S.I. units, $1eV = 1.6\text{x}10^{-19}$joules. The units $\text{MeV} = 10^6\text{eV}$, $\text{GeV} = 10^9\text{eV}$ and $\text{TeV} = 10^{12}\text{eV}$ are also often used. In terms of these units, atomic ionization energies are typically a few eV, nuclear binding energies are typically 8 MeV per particle, and the highest particle energies produced in present accelerators are of order 1 TeV.

In order to create a new particle of mass *M*, an energy at least as great as its rest energy Mc^2 must be supplied. The rest energies of the electron and proton are 0.51 MeV and 0.94 GeV respectively, whereas the *W* and Z^0 bosons have rest energies of 80 GeV and 91 GeV, respectively. Correspondingly their masses are conveniently measured in MeV/c^2 or GeV/c^2, so that, for example,

$$M_e = 0.51\text{MeV}/\text{c}^2,\ M_p = 0.94\ \text{GeV}/\text{c}^2,\ M_W = 80.3\ \text{GeV}/\text{c}^2,\ M_Z = 91.2\ \text{GeV}/\text{c}^2$$

In terms of S.I. units, $1\text{MeV}/\text{c}^2 = 1.78\text{x}10^{-30}\text{kg}$.

Although practical calculations are expressed in the above units, it is usual in particle physics to make theoretical calculations in units chosen such that $\hbar \equiv h/2\pi = 1$ and $c = 1$ (called *natural units*) and some books you meet will do this. However, in these lectures, as I will also be talking about nuclear physics, I will use only practical units.

Some useful conversion factors are:

$$\hbar = 6.58\text{x}10^{-25}\,\text{GeV.s}\ ; \qquad \hbar c = 1.97\text{x}10^{-16}\,\text{GeV.m}$$

You can find others in the recommended books.