

UNIVERSITY OF LONDON  
(University College London)

MSci Degree 2002

**PHYS3C24: NUCLEAR AND PARTICLE PHYSICS**

## Answer **THREE** questions

The numbers in square brackets at the right-hand edge of the paper indicate the provisional allocation of maximum marks for each subsection of a question.

**1.** Make a schematic sketch of a layered detector system suitable for studying  $e^+e^-$  annihilation reactions, labelling each sub-detector and indicating its role.

[6 marks]

$Z^0$  bosons are produced in  $e^+e^-$  collisions and then decay to pairs of fundamental fermions  $f\bar{f}$ . Draw the lowest-order Feynman diagram for the overall process.

[2 marks]

The couplings of the  $Z^0$  to right-handed (R) and left-handed (L) fermions are given by

$$g_R(f) = -q_f \sin^2 \theta_w, \quad g_L(f) = \pm 1/2 - q_f \sin^2 \theta_w$$

where  $q_f$  is the electric charge of the fermion  $f$  in units of  $e$  and  $\theta_w$  is the weak mixing angle. The positive sign in  $g_L$  is used for neutrinos and the  $q = u, c, t$  quarks; the negative sign is used for charged leptons and the  $q = d, s, b$  quarks. If the partial width for  $Z^0 \rightarrow f\bar{f}$  is given by

$$\Gamma_f = \frac{G_F M_Z^3 c^6}{3\pi\sqrt{2}(\hbar c)^3} [g_R^2(f) + g_L^2(f)]$$

calculate the partial widths to neutrinos  $\Gamma_\nu$  and to  $q\bar{q}$  pairs  $\Gamma_q$ .

[5 marks]

Explain the relation of  $\Gamma_q$  to the partial width to hadrons  $\Gamma_{had}$ .

[2 marks]

The widths to hadrons and to charged leptons are *measured* to be

$$\Gamma_{had} = (1738 \pm 12) \text{ MeV}, \quad \Gamma_{lep} = (250 \pm 2) \text{ MeV}$$

and the total width to all final states is *measured* to be

$$\Gamma_{tot} = (2490 \pm 7) \text{ MeV}.$$

Use these experimental results and your calculated value for the decay width to neutrinos to show that there are only three generations of neutrinos with masses  $M_\nu < M_Z/2$ .

[5 marks]

[Numerical values:  $G_F/(\hbar c)^3 = 1.17 \times 10^{-5} \text{ GeV}^{-2}$ ;  $M_Z = 91.2 \text{ GeV}/c^2$ ;  $\sin^2 \theta_w \approx 1/4$ ]

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**PLEASE TURN OVER**

**2.** The mass  $M$  of a nucleus with an odd value of  $A$ , the mass number, is given by the semi-empirical mass formula (SEMF)

$$M(Z, A) = Z m_p + (A - Z) m_n - a_v A + a_s A^{2/3} + a_c Z(Z - 1) A^{-1/3} + a_a (Z - A/2)^2 A^{-1}$$

where  $Z$  is the number of protons,  $m_p$  is the mass of the proton,  $m_n$  is the mass of the neutron and the constants  $a_i > 0$ . Explain briefly the physical significance and form of each of the terms in this expression.

**[5 marks]**

Write down an expression for the binding energy  $B$  of the nucleus in terms of its rest mass and the rest masses of its constituent nucleons. Use the SEMF to show that  $B$  is a quadratic in  $Z$  for a fixed value of  $A$ , and hence find the value of  $Z$  for the most stable nucleus with  $A = 101$ .

**[4 marks]**

Explain, without detailed calculation, how nuclides with the same  $A$ , but with values of  $Z$  not corresponding to the most stable configuration can convert to nuclei of greater stability. In each case state the condition on the atomic masses involved for this to be possible.

**[6 marks]**

Assume that in the shell model the nucleon energy levels are ordered

$$1s, 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, \dots$$

State the configuration of the ground state of  ${}^7_3\text{Li}$  and give the two most likely configurations for the first excited state, assuming that only protons are excited. In each case explain the reasons for your choice.

**[5 marks]**

[Numerical values:  $a_c = 0.714 \text{ MeV}/c^2$ ;  $a_a = 93.15 \text{ MeV}/c^2$ ]

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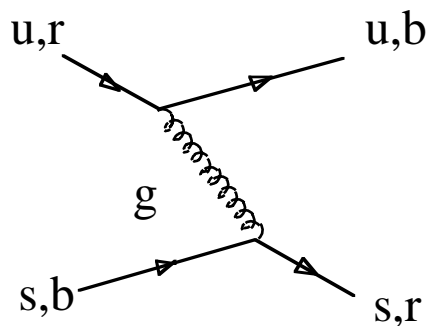
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**3.** State the postulates of the simple quark model of hadrons. **[3 marks]**

Why does the existence of the low-lying baryon state  $\Omega^- = sss$  (where  $s$  is a strange quark) with spin  $J = 3/2$  imply that quarks must possess the attribute called colour? **[3 marks]**

Consider all the lowest-lying baryon states that can be constructed in the simple quark model from the quarks  $q = u, d$  and  $s$ . By considering the possible spin states, deduce that there are eight with spin  $J = 1/2$  and ten with  $J = 3/2$ . **[6 marks]**

The lowest-order Feynman diagram for the scattering of a red  $u$ -quark with a blue  $s$ -quark by gluon exchange is given below.



What does the existence of this diagram imply about the gluon-gluon interaction and how does this differ from the interaction of photons? Illustrate your answer using Feynman diagrams. **[3 marks]**

Discuss briefly, the consequences of this difference for the dependence of the strong interaction coupling  $\alpha_s$  with distance, comparing the situation with that of the electromagnetic interaction. Use Feynman diagrams to illustrate your answer. **[5 marks]**

**PLEASE TURN OVER**

4. What is the main property of particles that is measured in a calorimeter? Why does a calorimeter differ from most other detectors and give one reason why this device is an important class of detector in particle physics.

[2 marks]

Describe briefly the physical processes whereby electrons are detected in an electromagnetic calorimeter. Give one major difference between the latter detector and a hadronic calorimeter.

[3 marks]

Define charged current and neutral current reactions in weak interactions and give an example of each in symbol form. How do they differ in respect of conservation of the strangeness quantum number?

[5 marks]

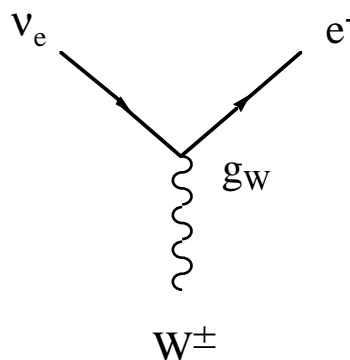
Use the lepton-quark substitutions

$$\nu_e \rightarrow u, \quad e^- \rightarrow d'$$

where

$$d' = d \cos \theta_C + s \sin \theta_C$$

and  $\theta_C$  is the Cabibbo angle, to derive the  $W^\pm$ -quark vertices and their strengths corresponding to the  $W^\pm$ -lepton vertices:



[3 marks]

Draw the lowest-order Feynman diagrams for the decays [ $\pi^- = d\bar{u}$ ;  $K^- = s\bar{u}$ ]

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad \text{and} \quad K^- \rightarrow \mu^- + \bar{\nu}_\mu .$$

[4 marks]

Using  $\theta_C = 12^\circ$ , and ignoring all kinematic and spin effects, estimate the ratio

$$R \equiv \frac{\text{Rate}(K^- \rightarrow \mu^- + \bar{\nu}_\mu)}{\text{Rate}(\pi^- \rightarrow \mu^- + \bar{\nu}_\mu)} .$$

[3 marks]

CONTINUED

**5.** Define the term *spontaneous fission* and explain why this a rare process. If the mean energy of fission neutrons in  $^{235}_{92}\text{U}$  is 2 MeV and 75% of the fission neutrons in  $^{238}_{92}\text{U}$  have an energy greater than 1.4 MeV, explain why spontaneous fission of natural uranium does not lead to a chain reaction, even though on average 2.2 neutrons are produced in each fission?

**[5 marks]**

Define the term *induced fission* and explain why thermal neutrons can induce fission in  $^{235}_{92}\text{U}$  but not in  $^{238}_{92}\text{U}$

**[3 marks]**

A core of a thermal nuclear reactor consists of control rods, fuel elements and a moderator. Sketch a schematic diagram of the core and discuss the role of each of these parts.

**[4 marks]**

In a particular neutron-induced fission reaction of  $^{235}_{92}\text{U}$ , the fission fragments  $^{92}_{37}\text{Rb}$  and  $^{140}_{55}\text{Cs}$  are produced with total energy released of 166 MeV. If this reaction were used to power a 100 MW [ $1\text{MeV} = 1.6 \times 10^{-13} \text{ J}$ ] nuclear reactor whose core is a sphere of radius 100 cm and an average of 1 neutron per fission escapes the core, what is the neutron flux at the outer surface of the reactor in  $\text{s}^{-1}\text{m}^{-2}$ ?

**[4 marks]**

Explain the difference between *prompt* and *delayed* neutrons in fission reactions and the role of the latter in nuclear reactors.

**[4 marks]**

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**END OF PAPER**